



# SIMPLE HARMONIC MOTION



## 1. PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular interval of time, its motion is said to be **Periodic Motion** and interval of time is called **time period** or harmonic motion period (T). The path of periodic motion may be linear, circular, elliptical or any other curve. For example, rotation of earth about the sun.

## 2. OSCILLATORY MOTION

'To and Fro' type of motion is called an **Oscillatory Motion**. It need not be periodic and need not have fixed extreme positions. For example, motion of pendulum of a wall clock.

The oscillatory motions in which energy is conserved are also periodic.

The force / torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force / torque.

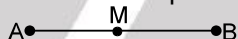
**Damped oscillations** are those in which energy is consumed due to some resistive forces and hence total mechanical energy decreases.

## 3. SIMPLE HARMONIC MOTION

If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called simple Harmonic Motion (SHM). It is the simplest (easy to analyze) form of oscillatory motion.

### 3.1 TYPES OF SHM

(a) **Linear SHM** : When a particle undergoes to and fro motion about an equilibrium position, along a straight line. A and B are extreme positions. M is mean position.  $AM = MB = \text{Amplitude}$



(b) **Angular SHM** : When a body/particle is free to rotate oscillate about a given axis on a curved path.

### 3.2 EQUATION OF SIMPLE HARMONIC MOTION (SHM) :

The necessary and sufficient condition for SHM is  $F = -kx$

where  $k$  = positive constant for a SHM = Force constant

$x$  = displacement from mean position.

$$\text{or } m \frac{d^2x}{dt^2} = -kx$$

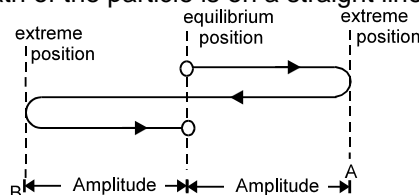
$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad [\text{differential equation of SHM}]$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2x = 0 \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

It's solution is  $x = A \sin(\omega t + \phi)$

### 3.3 CHARACTERISTICS OF SHM

**Note** : In the figure shown, path of the particle is on a straight line.



(a) **Displacement** - It is defined as the distance of the particle from the mean position at that instant.

Displacement in SHM at time  $t$  is given by  $x = A \sin(\omega t + \phi)$



(b) **Amplitude** : It is the maximum value of displacement of the particle from its equilibrium position.

$$\text{Amplitude} = \frac{1}{2} [\text{distance between extreme points or positions}]$$

It depends on energy of the system.

(c) **Angular Frequency ( $\omega$ )** :  $\omega = \frac{2\pi}{T} = 2\pi f$  and its unit is rad/sec.

(d) **Frequency ( $f$ )** : Number of oscillations completed in unit time interval is called frequency of oscillations,  $f = \frac{1}{T} = \frac{\omega}{2\pi}$ , its units is  $\text{sec}^{-1}$  or Hz.

(e) **Time period ( $T$ )** : Smallest time interval after which the oscillatory motion gets repeated is called time period,  $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

## Solved Examples

**Example 1.** For a particle performing SHM, equation of motion is given as  $\frac{d^2x}{dt^2} + 4x = 0$ . Find the time period.

**Solution :**  $\frac{d^2x}{dt^2} = -4x$        $\omega^2 = 4$        $\omega = 2$

Time period ;  $T = \frac{2\pi}{\omega} = \pi$



(f) **Phase** : The physical quantity which represents the state of motion of particle (eg. its position and direction of motion at any instant).

The argument ( $\omega t + \phi$ ) of sinusoidal function is called instantaneous phase of the motion.

(g) **Phase constant ( $\phi$ )** : Constant  $\phi$  in equation of SHM is called phase constant or initial phase. It depends on initial position and direction of velocity.

(h) **Velocity( $v$ )** : Velocity at an instant is the rate of change of particle's position w.r.t time at that instant.

Let the displacement from mean position is given by

$$x = A \sin(\omega t + \phi)$$

Velocity,  $v = \frac{dx}{dt} = \frac{d}{dt}[A \sin(\omega t + \phi)]$

$$v = A\omega \cos(\omega t + \phi) \quad \text{or,} \quad v = \omega\sqrt{A^2 - x^2}$$

At mean position ( $x = 0$ ), velocity is maximum.

$$v_{\max} = \omega A$$

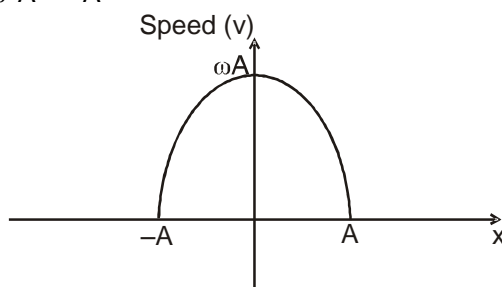
At extreme position ( $x = A$ ), velocity is minimum.

$$v_{\min} = \text{zero}$$

### GRAPH OF SPEED ( $v$ ) VS DISPLACEMENT ( $x$ ):

$$v = \omega\sqrt{A^2 - x^2} \quad v^2 = \omega^2(A^2 - x^2)$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2 \quad \frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$



**GRAPH WOULD BE AN ELLIPSE**

- (i) **Acceleration** : Acceleration at an instant is the rate of change of particle's velocity w.r.t. time at that instant.

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}[A\omega \cos(\omega t + \phi)]$$

$$a = -\omega^2 A \sin(\omega t + \phi)$$

$$a = -\omega^2 x$$

**Note**

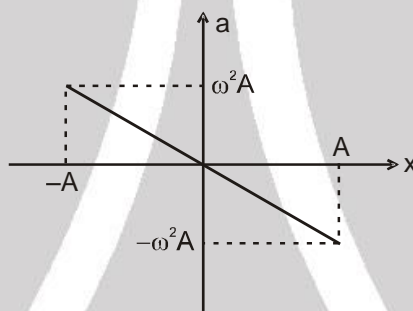
- Negative sign shows that acceleration is always directed towards the mean position.

At mean position ( $x = 0$ ), acceleration is minimum.

$$a_{\min} = \text{zero}$$

At extreme position ( $x = A$ ), acceleration is maximum.

$$a_{\max} = \omega^2 A$$

**GRAPH OF ACCELERATION (A) VS DISPLACEMENT (x)**

$$a = -\omega^2 x$$

**Solved Examples**

- Example 2.** The equation of particle executing simple harmonic motion is  $x = (5 \text{ m}) \sin \left[ (\pi \text{ s}^{-1})t + \frac{\pi}{3} \right]$ . Write down the amplitude, time period and maximum speed. Also find the velocity at  $t = 1 \text{ s}$ .

**Solution :** Comparing with equation  $x = A \sin(\omega t + \delta)$ , we see that the amplitude = 5 m,

$$\text{and time period} = \frac{2\pi}{\omega} = \frac{2\pi}{\pi \text{ s}^{-1}} = 2 \text{ s}.$$

$$\text{The maximum speed} = A\omega = 5 \text{ m} \times \pi \text{ s}^{-1} = 5\pi \text{ m/s}.$$

$$\text{The velocity at time } t = \frac{dx}{dt} = A\omega \cos(\omega t + \delta)$$

At  $t = 1 \text{ s}$ ,

$$v = (5 \text{ m}) (\pi \text{ s}^{-1}) \cos \left( \pi + \frac{\pi}{3} \right) = -\frac{5\pi}{2} \text{ m/s}.$$





**Example 3.** A particle executing simple harmonic motion has angular frequency  $6.28 \text{ s}^{-1}$  and amplitude  $10 \text{ cm}$ . Find (a) the time period, (b) the maximum speed, (c) the maximum acceleration, (d) the speed when the displacement is  $6 \text{ cm}$  from the mean position, (e) the speed at  $t = 1/6 \text{ s}$  assuming that the motion starts from rest at  $t = 0$ .

**Solution :**

(a) Time period  $= \frac{2\pi}{\omega} = \frac{2\pi}{6.28} \text{ s} = 1 \text{ s}$ .

(b) Maximum speed  $= A\omega = (0.1 \text{ m}) (6.28 \text{ s}^{-1}) = 0.628 \text{ m/s}$ .

(c) Maximum acceleration  $= A\omega^2$   
 $= (0.1 \text{ m}) (6.28 \text{ s}^{-1})^2 = 4 \text{ m/s}^2$ .

(d)  $v = \omega \sqrt{A^2 - x^2} = (6.28 \text{ s}^{-1}) \sqrt{(10\text{cm})^2 - (6\text{cm})^2} = 50.2 \text{ cm/s}$ .

(e) At  $t = 0$ , the velocity is zero i.e., the particle is at an extreme. The equation for displacement may be written as

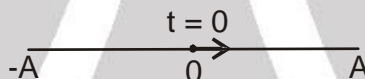
$$x = A \cos \omega t.$$

$$\text{The velocity is } v = -A \omega \sin \omega t.$$

$$\text{At } t = \frac{1}{6} \text{ s}, v = -(0.1 \text{ m}) (6.28 \text{ s}^{-1}) \sin\left(\frac{6.28}{6}\right)$$

$$= (-0.628 \text{ m/s}) \sin \frac{\pi}{3} = 54.4 \text{ cm/s}.$$

**Example 4.** A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is  $A$ .



**Solution :** General equation of SHM can be written as  $x = A \sin(\omega t + \phi)$

$$\text{At } t = 0, x = 0$$

$$\therefore 0 = A \sin \phi$$

$$\therefore \phi = 0, \pi$$

$$\phi \in [0, 2\pi)$$

Also; at  $t = 0, v = +ve$

$$\therefore A\omega \cos \phi = +ve$$

$$\text{or, } \phi = 0$$

Hence, if the particle is at mean position at  $t = 0$  and is moving towards +ve extreme, then the equation of SHM is given by  $x = A \sin \omega t$

**Similarly**

for

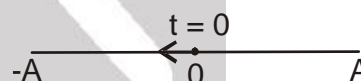
$$\phi = \pi$$

$$\therefore \text{equation of SHM is } x = A \sin(\omega t + \pi)$$

$$\text{or, } x = -A \sin \omega t$$

**Note :**

- If mean position is not at the origin, then we can replace  $x$  by  $x - x_0$  and the eqn. becomes  $x - x_0 = A \sin(\omega t + \phi)$ , where  $x_0$  is the position co-ordinate of the mean position.



**Example 5.** A particle is performing SHM of amplitude " $A$ " and time period " $T$ ". Find the time taken by the particle to go from  $0$  to  $A/2$ .

**Solution :** Let equation of SHM be  $x = A \sin \omega t$

$$\text{when } x = 0, t = 0$$

$$\text{when } x = A/2; A/2 = A \sin \omega t$$

$$\text{or } \sin \omega t = 1/2 \quad \omega t = \pi/6$$

$$\frac{2\pi}{T} t = \pi/6 \quad t = T/12$$

Hence, time taken is  $T/12$ , where  $T$  is time period of SHM.



**Example 6.** A particle of mass 2 kg is moving on a straight line under the action of force  $F = (8 - 2x)$  N. It is released at rest from  $x = 6$  m.

- Is the particle moving simple harmonically.
- Find the equilibrium position of the particle.
- Write the equation of motion of the particle.
- Find the time period of SHM.

**Solution :**

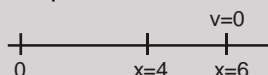
$$F = 8 - 2x \quad \text{or} \quad F = -2(x - 4)$$

at equilibrium position  $F = 0$

$$\Rightarrow x = 4 \text{ is equilibrium position}$$

Hence the motion of particle is SHM with force constant 2 and equilibrium position  $x = 4$ .

- Yes, motion is SHM.
- Equilibrium position is  $x = 4$



- At  $x = 6$  m, particle is at rest i.e. it is one of the extreme position. Hence amplitude is  $A = 2$  m and initially particle is at the extreme position.  $\therefore$  Equation of SHM can be written as

$$x - 4 = 2 \cos \omega t, \quad \text{where } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1$$

$$\text{i.e. } x = 4 + 2 \cos t$$

- Time period,  $T = \frac{2\pi}{\omega} = 2\pi \text{ sec.}$



#### 4. SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle moving on a circle of radius  $A$  with a constant angular speed  $\omega$  as shown in figure.

Suppose the particle is on the top of the circle (Y-axis) at  $t = 0$ . The radius  $OP$  makes an angle  $\theta = \omega t$  with the Y-axis at time  $t$ . Drop a perpendicular  $PQ$  on X-axis. The components of position vector, velocity vector and acceleration vector at time  $t$  on the X-axis are

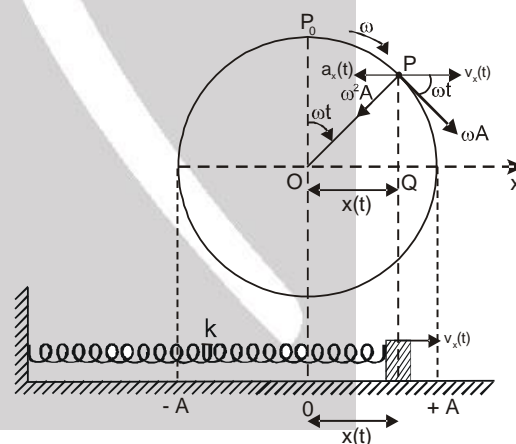
$$x(t) = A \sin \omega t$$

$$v_x(t) = A\omega \cos \omega t$$

$$a_x(t) = -\omega^2 A \sin \omega t$$

Above equations show that the foot of perpendicular  $Q$  executes a simple harmonic motion on the X-axis. The amplitude is  $A$  and angular frequency is  $\omega$ .

Similarly the foot of perpendicular on Y-axis will also execute SHM of amplitude  $A$  and angular frequency  $\omega$  [ $y(t) = A \cos \omega t$ ]. The phases of the two simple harmonic motions differ by  $\pi/2$ .



#### 5. GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY & ACCELERATION IN SHM

Displacement,  $x = A \sin \omega t$

Velocity,  $v = A\omega \cos \omega t = A\omega \sin \left( \omega t + \frac{\pi}{2} \right)$  or  $v = \sqrt{A^2 - x^2} \omega$

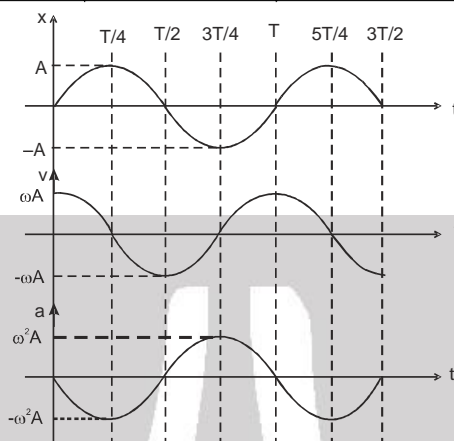
Acceleration,  $a = -\omega^2 A \sin \omega t = \omega^2 A \sin (\omega t + \pi)$  or  $a = -\omega^2 x$

**Note :** •  $v = \omega \sqrt{A^2 - x^2}$   
 $a = -\omega^2 x$

These relations are true for any equation of  $x$ .



time, t	0	T/4	T/2	3T/4	T
displacement, x	0	A	0	-A	0
Velocity, v	$A\omega$	0	$-A\omega$	0	$A\omega$
acceleration, a	0	$-\omega^2 A$	0	$\omega^2 A$	0



1. All the three quantities displacement, velocity and acceleration vary harmonically with time, having same period.
2. The velocity amplitude is  $\omega$  times the displacement amplitude ( $v_{\max} = \omega A$ ).
3. The acceleration amplitude is  $\omega^2$  times the displacement amplitude ( $a_{\max} = \omega^2 A$ ).
4. In SHM, the velocity is ahead of displacement by a phase angle of  $\frac{\pi}{2}$ .
5. In SHM, the acceleration is ahead of velocity by a phase angle of  $\frac{\pi}{2}$ .



## 6. ENERGY OF SHM

### 6.1 Kinetic Energy (KE)

$$\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k (A^2 - x^2) \text{ (as a function of } x\text{)}$$

$$= \frac{1}{2} m A^2 \omega^2 \cos^2 (\omega t + \theta) = \frac{1}{2} K A^2 \cos^2 (\omega t + \theta) \text{ (as a function of } t\text{)}$$

$$KE_{\max} = \frac{1}{2} k A^2 \quad ; \quad \langle KE \rangle_{0-T} = \frac{1}{4} k A^2 \quad ; \quad \langle KE \rangle_{0-A} = \frac{1}{3} k A^2$$

Frequency of KE = 2 × (frequency of SHM)

### 6.2 Potential Energy (PE)

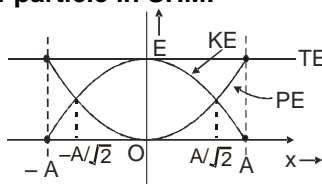
$$\frac{1}{2} Kx^2 \text{ (as a function of } x\text{)} = \frac{1}{2} k A^2 \sin^2 (\omega t + \theta) \text{ (as a function of time)}$$

### 6.3 Total Mechanical Energy (TME)

$$\text{Total mechanical energy} = \text{Kinetic energy} + \text{Potential energy} = \frac{1}{2} k(A^2 - x^2) + \frac{1}{2} Kx^2 = \frac{1}{2} K A^2$$

Hence total mechanical energy is constant in SHM.

### 6.4 Graphical Variation of energy of particle in SHM.





## Solved Examples

**Example 7.** A particle of mass 0.50 kg executes a simple harmonic motion under a force  $F = - (50 \text{ N/m})x$ . If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of the motion.

**Solution :** The kinetic energy of the particle when it is at the centre of oscillation is

$$E = \frac{1}{2} mv^2 = \frac{1}{2} (0.50 \text{ kg}) (10 \text{ m/s})^2 = 25 \text{ J}.$$

The potential energy is zero here. At the maximum displacement  $x = A$ , the speed is zero and hence the kinetic energy is zero. The potential energy here is  $\frac{1}{2} kA^2$ . As there is no loss of energy,

$$\frac{1}{2} kA^2 = 25 \text{ J} \quad \text{.....(i)}$$

The force on the particle is given by

$$F = - (50 \text{ N/m})x.$$

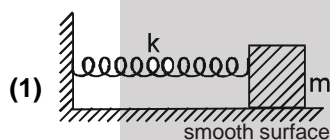
Thus, the spring constant is  $k = 50 \text{ N/m}$ .

Equation (i) gives

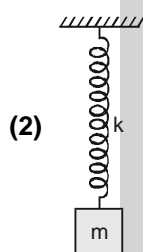
$$\frac{1}{2} (50 \text{ N/m}) A^2 = 25 \text{ J} \quad \text{or,} \quad A = 1 \text{ m}.$$



## 7. SPRING-MASS SYSTEM

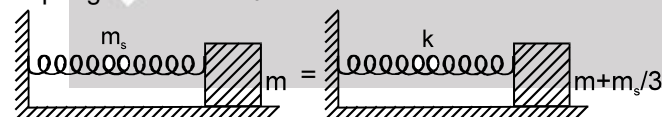


$$T = 2\pi \sqrt{\frac{m}{k}}$$



$$T = 2\pi \sqrt{\frac{m}{k}}$$

(3) If spring has mass  $m_s$  then



$$T = 2\pi \sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

## Solved Examples

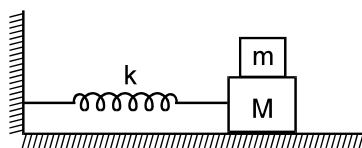
**Example 8.** A particle of mass 200 g executes a simple harmonic motion. The restoring force is provided by a spring of spring constant 80 N/m. Find the time period.

**Solution :** The time period is  $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{200 \times 10^{-3} \text{ kg}}{80 \text{ N/m}}} = 2\pi \times 0.05 \text{ s} = 0.31 \text{ s}.$





**Example 9.** The friction coefficient between the two blocks shown in figure is  $\mu$  and the horizontal plane is smooth. (a) If the system is slightly displaced and released, find the time period. (b) Find the magnitude of the frictional force between the blocks when the displacement from the mean position is  $x$ . (c) What can be the maximum amplitude if the upper block does not slip relative to the lower block ?



**Solution :** (a) For small amplitude, the two blocks oscillate together. The angular frequency is

$$\omega = \sqrt{\frac{k}{M+m}} \text{ and so the time period } T = 2\pi \sqrt{\frac{M+m}{k}}.$$

(b) The acceleration of the blocks at displacement  $x$  from the mean position is

$$a = -\omega^2 x = \left( \frac{-kx}{M+m} \right)$$

The resultant force on the upper block is, therefore,  $ma = \left( \frac{-mkx}{M+m} \right)$

This force is provided by the friction of the lower block. Hence, the magnitude of the frictional force is  $\left( \frac{mk |x|}{M+m} \right)$

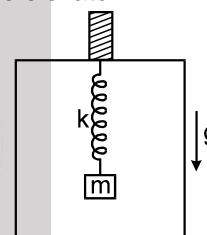
(c) Maximum force of friction required for simple harmonic motion of the upper block is  $\frac{mkA}{M+m}$  at the extreme positions. But the maximum frictional force can only be  $\mu mg$ . Hence

$$\frac{mk}{M+m} A = \mu mg \quad \text{or,} \quad A = \frac{\mu(M+m)g}{k}$$

**Example 10.** A block of mass  $m$  is suspended from the ceiling of a stationary elevator through a spring of spring constant  $k$  and suddenly, the cable breaks and the elevator starts falling freely. Show that block now executes a simple harmonic motion of amplitude  $mg/k$  in the elevator.

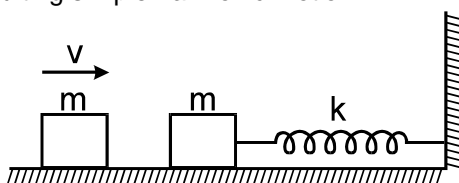
**Solution :** When the elevator is stationary, the spring is stretched to support the block. If the extension is  $x$ , the tension is  $kx$  which should balance the weight of the block.

Thus,  $x = mg/k$ . As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a pseudo force  $mg$  upward on the block. This force will 'balance' the weight.



Thus, the block is subjected to a net force  $kx$  by the spring when it is at a distance  $x$  from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by  $x = mg/k$ , where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is  $mg/k$ .

**Example 11.** The left block in figure collides inelastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.







**Solution :** Assuming the collision to last for a small interval only, we can apply the principle of conservation of momentum. The common velocity after the collision is  $\frac{v}{2}$ .

$$\text{The kinetic energy} = \frac{1}{2} (2m) \left( \frac{v}{2} \right)^2 = \frac{1}{4} mv^2.$$

This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is  $A$ , the total energy can also be written as  $\frac{1}{2} kA^2$ . Thus,

$$\frac{1}{2} kA^2 = \frac{1}{4} mv^2, \text{ giving } A = \sqrt{\frac{m}{2k}} v.$$

**Example 12.** Two blocks of mass  $m_1$  and  $m_2$  are connected with a spring of natural length  $l$  and spring constant  $k$ . The system is lying on a smooth horizontal surface. Initially spring is compressed by  $x_0$  as shown in figure.

Show that the two blocks will perform SHM about their equilibrium position. Also (a) find the time period, (b) find amplitude of each block and (c) length of spring as a function of time.

**Solution :** (a) Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let  $EP_1$  and  $EP_2$  be equilibrium positions of block A and B as shown in figure.

Let at any time during oscillations, blocks are at a distance of  $x_1$  and  $x_2$  from their equilibrium positions. As no external force is acting on the spring block system

$$\therefore (m_1 + m_2) \Delta x_{cm} = m_1 x_1 - m_2 x_2 = 0 \quad \text{or}$$

For 1st particle, force equation can be written as

$$k(x_1 + x_2) = -m_1 \frac{d^2 x_1}{dt^2} \quad \text{or,} \quad k\left(x_1 + \frac{m_1}{m_2} x_1\right) = -m_1 a_1$$

$$\text{or, } a_1 = \frac{k(m_1 + m_2)}{m_1 m_2} x_1 \quad \therefore \omega^2 = \frac{k(m_1 + m_2)}{m_1 m_2}$$

$$\text{Hence, } T = 2\pi \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} = 2\pi \sqrt{\frac{\mu}{K}}$$

where  $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$  which is known as reduced mass

**Ans (a)**

Similarly time period of 2nd particle can be found. Both will be having the same time period.

(b) Let the amplitude of blocks be  $A_1$  and  $A_2$ .

$$m_1 A_1 = m_2 A_2$$

$$\text{By energy conservation; } \frac{1}{2} k(A_1 + A_2)^2 = \frac{1}{2} k x_0^2 \quad \text{or,} \quad A_1 + A_2 = x_0$$

$$\text{or, } A_1 + A_2 = x_0 \quad \text{or,} \quad A_1 + \frac{m_1}{m_2} A_1 = x_0$$

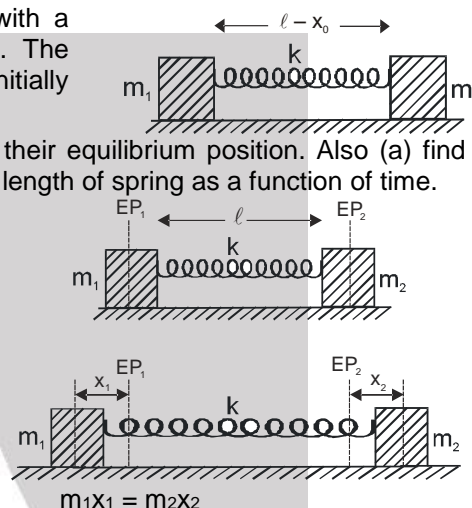
$$\text{or, } A_1 = \frac{m_2 x_0}{m_1 + m_2} \quad \text{Similarly,} \quad A_2 = \frac{m_1 x_0}{m_1 + m_2}$$

(c) Consider equilibrium position of 1st particle as origin, i.e.  $x = 0$ .  $x$  co-ordinate of particles can be written as

$$x_1 = A_1 \cos \omega t \quad \text{and} \quad x_2 = \ell - A_2 \cos \omega t$$

Hence, length of spring at time  $t$  can be written as;

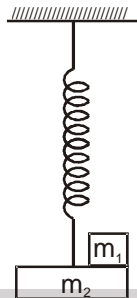
$$\text{length} = x_2 - x_1 = \ell - (A_1 + A_2) \cos \omega t$$





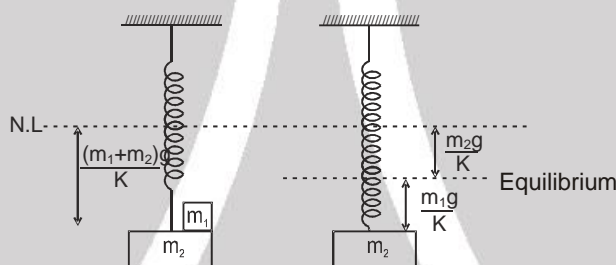
## Solved Examples

**Example 13.** The system is in equilibrium and at rest. Now mass  $m_1$  is removed from  $m_2$ . Find the time period and amplitude of resultant motion. Spring constant is  $K$ .



**Solution :** Initial extension in the spring  $x = \frac{(m_1 + m_2)g}{K}$

Now, if we remove  $m_1$  equilibrium position (E.P.) of  $m_2$  will be  $\frac{m_2 g}{K}$  below natural length of spring.



At the initial position, since velocity is zero i.e. it is the extreme position.

$$\text{Hence Amplitude} = \frac{m_1 g}{K}$$

$$\text{Time period} = 2\pi \sqrt{\frac{m_2}{K}}$$

Since only block of mass  $m_2$  is oscillating



## 8. COMBINATION OF SPRINGS

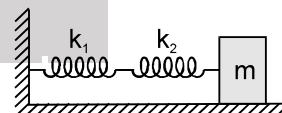
### 8.1 Series Combination :

Total displacement  $x = x_1 + x_2$

Tension in both springs  $= k_1 x_1 = k_2 x_2$

$\therefore$  Equivalent spring constant in series combination  $K_{eq}$  is given by :

$$\frac{1}{K_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow T = 2\pi \sqrt{\frac{m}{K_{eq}}}$$



### Note :

- In series combination, tension is same in all the springs & extension will be different. (If  $k$  is same then deformation is also same)
- In series combination, extension of springs will be reciprocal of its spring constant.
- Spring constant of spring is reciprocal of its natural length  
 $\therefore k \propto 1/l$   
 $\therefore k_1 l_1 = k_2 l_2 = k_3 l_3$
- If a spring is cut in 'n' pieces then spring constant of one piece will be  $nk$ .

### 8.2 Parallel combination :



**Resonance**  
Educating for better tomorrow

Corp. / Reg. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005  
 Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)  
 Toll Free : 1800 258 5555 | CIN : U80302RJ2007PLC024029

ADVSH - 10

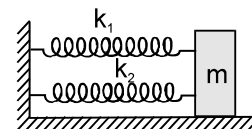


Extension is same for both springs but force acting will be different.

Force acting on the system = F

$$\therefore F = -(k_1 x + k_2 x) \Rightarrow F = -(k_1 + k_2) x \Rightarrow F = -k_{eq} x$$

$$\therefore k_{eq} = k_1 + k_2 \Rightarrow T = 2\pi \sqrt{\frac{m}{k_{eq}}}$$



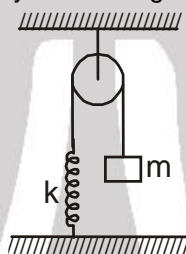
## 9. METHOD'S TO DETERMINE TIME PERIOD, ANGULAR FREQUENCY IN S.H.M.

(a) Force / torque method

(b) Energy method

### Solved Examples

**Example 14.** The string, the spring and the pulley shown in figure are light. Find the time period of the mass m.



**Solution :**

**(a) Force Method :**

Let in equilibrium position of the block, extension in spring is  $x_0$ .

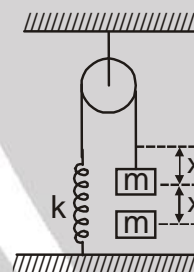
$$\therefore kx_0 = mg \quad \dots(1)$$

Now if we displace the block by  $x$  in the downward direction, net force on the block towards mean position is

$$F = k(x + x_0) - mg = kx \quad \text{using (1)}$$

Hence the net force is acting towards mean position and is also proportional to  $x$ .

So, the particle will perform S.H.M. and its time period would be  $T = 2\pi \sqrt{\frac{m}{k}}$



**(b) Energy Method :**

Let gravitational potential energy is to be zero at the level of the block when spring is in its natural length.

Now at a distance  $x$  below that level, let speed of the block be  $v$ .

Since total mechanical energy is conserved in S.H.M.

$$\therefore -mgx + \frac{1}{2} kx^2 + \frac{1}{2} mv^2 = \text{constant}$$

Differentiating w.r.t. time, we get

$$-mgv + kxv + mva = 0$$

where  $a$  is acceleration.

$$\therefore F = ma = -kx + mg \quad \text{or} \quad F = -k\left(x - \frac{mg}{k}\right)$$

This shows that for the motion, force constant is  $k$  and equilibrium position is  $x = \frac{mg}{k}$ .

So, the particle will perform S.H.M. and its time period would be  $T = 2\pi \sqrt{\frac{m}{k}}$





## 10. SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

Time period of a simple pendulum  $T = 2\pi \sqrt{\frac{\ell}{g}}$

(some times we can take  $g = \pi^2$  for making calculation simple)

**Note :**

- If angular amplitude of simple pendulum is more, then time period

$$T = 2\pi \sqrt{\frac{\ell}{g}} \left( 1 + \frac{\theta_0^2}{16} \right)$$

(Not in JEE, For other exams)

where  $\theta_0$  is in radians.

- General formula for time period of simple pendulum when  $\ell$  is comparable to radius of Earth R.

$$T = 2\pi \sqrt{\frac{1}{g \left( \frac{1}{R} + \frac{1}{\ell} \right)}}$$

where, R = Radius of the earth

- Time period of simple pendulum of infinite length is maximum and is given by :  $T = 2\pi \sqrt{\frac{R}{g}} = 84.6 \text{ min}$

(Where R is radius of earth)

- Time period of seconds pendulum is 2 sec and  $\ell = 0.993 \text{ m}$ .
- Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as linear S.H.M.
- If time period of clock based on simple pendulum increases then clock will be slow but if time period decrease then clock will be fast.

- If g remains constant &  $\Delta \ell$  is change in length, then  $\frac{\Delta T}{T} \times 100 = \frac{1}{2} \frac{\Delta \ell}{\ell} \times 100$

- If  $\ell$  remain constant &  $\Delta g$  is change in acceleration then,  $\frac{\Delta T}{T} \times 100 = -\frac{1}{2} \frac{\Delta g}{g} \times 100$

- If  $\Delta \ell$  is change in length &  $\Delta g$  is change in acceleration due to gravity then,

$$\frac{\Delta T}{T} \times 100 = \left[ \frac{1}{2} \frac{\Delta \ell}{\ell} - \frac{1}{2} \frac{\Delta g}{g} \right] \times 100$$

### Solved Examples

**Example 15** A simple pendulum of length 40 cm oscillates with an angular amplitude of 0.04 rad. Find (a) the time period, (b) the linear amplitude of the bob, (c) the speed of the bob when the string makes 0.02 rad with the vertical and (d) the angular acceleration when the bob is in momentary rest. Take  $g = 10 \text{ m/s}^2$ .

**Solution :** (a) The angular frequency is  $\omega = \sqrt{g/\ell} = \sqrt{\frac{10 \text{ m/s}^2}{0.4 \text{ m}}} = 5 \text{ s}^{-1}$

the time period is  $\frac{2\pi}{\omega} = \frac{2\pi}{5 \text{ s}^{-1}} = 1.26 \text{ s}$ .

(b) Linear amplitude =  $40 \text{ cm} \times 0.04 = 1.6 \text{ cm}$

(c) Angular speed at displacement 0.02 rad is

$$\Omega = (5 \text{ s}^{-1}) \sqrt{(0.04)^2 - (0.02)^2} \text{ rad} = 0.17 \text{ rad/s.}$$

where speed of the bob at this instant =  $(40 \text{ cm}) \times 0.175 \text{ s}^{-1} = 6.8 \text{ cm/s}$ .

(d) At momentary rest, the bob is in extreme position.

Thus, the angular acceleration  $\alpha = (0.04 \text{ rad}) (25 \text{ s}^{-2}) = 1 \text{ rad/s}^2$ .



### 10.1 Time Period of Simple Pendulum in accelerating Reference Frame :



**Resonance®**  
Educating for better tomorrow

Corp. / Reg. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)

Toll Free : 1800 258 5555 | CIN : U80302RJ2007PLC024029

ADVSH - 12



$$T = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} \text{ where}$$

$g_{\text{eff}}$  = Effective acceleration in accelerating reference system =  $|\vec{g} - \vec{a}|$ , at mean position

$\vec{a}$  = acceleration of the point of suspension w.r.t. ground.

**Condition for applying this formula:**  $|\vec{g} - \vec{a}| = \text{constant}$

Also  $g_{\text{eff}} = \frac{\text{Net tension in string}}{\text{mass of bob}}$  at mean position

## Solved Examples

**Example 16.** A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is  $a_0$  and the length of the pendulum is  $\ell$ , find the time period of small oscillations about the mean position.

**Solution :** We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force  $ma_0$  on the bob of mass  $m$ .

For mean position, the acceleration of the bob with respect to the car should be zero. If  $\theta_0$  be the angle made by the string with the vertical, the tension, weight and the pseudo force will add to zero in this position.

Hence, resultant of  $mg$  and  $ma_0$  (say  $F = m\sqrt{g^2 + a_0^2}$ ) has to be along the string.

$$\therefore \tan \theta_0 = \frac{ma_0}{mg} = \frac{a_0}{g}$$

Now, suppose the string is further deflected by an angle  $\theta$  as shown in figure.

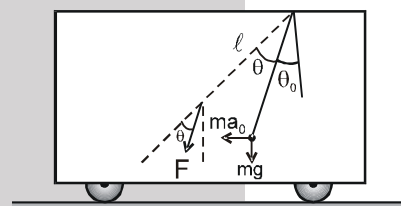
Now, restoring torque can be given by

$$(F \sin \theta) = \ell - m\ell^2 \alpha$$

Substituting  $F$  and using  $\sin \theta \simeq \theta$ , for small  $\theta$ .

$$(m\sqrt{g^2 + a_0^2})\ell \theta = -m\ell^2 \alpha$$

$$\text{or, } \alpha = \frac{\sqrt{g^2 + a_0^2}}{\ell} \theta \quad \text{so; } \omega^2 = \frac{\sqrt{g^2 + a_0^2}}{\ell}$$



This is an equation of simple harmonic motion with time period  $T = \frac{2\pi}{\omega} = 2\pi \frac{\sqrt{\ell}}{(g^2 + a_0^2)^{1/4}}$



**10.2** If forces other than  $m\vec{g}$  acts then :

$$T = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} \text{ where } g_{\text{eff}} = \left| \vec{g} + \frac{\vec{F}}{m} \right|$$

$\vec{F}$  = constant force acting on 'm'.

## Solved Examples

**Example 17.** A simple pendulum of length ' $\ell$ ' and having bob of mass ' $m$ ' is doing angular SHM inside water. A constant buoyant force equal to half the weight of the bob is acting on the ball. Find the time period of oscillations?

**Solution :** Here  $g_{\text{eff}} = g - \frac{mg/2}{m} = g/2$ . Hence  $T = 2\pi \sqrt{\frac{2\ell}{g}}$



## 11. COMPOUND PENDULUM / PHYSICAL PENDULUM



When a rigid body is suspended from an axis and made to oscillate about that then it is called compound pendulum.

C = Position of center of mass

S = Point of suspension

$\ell$  = Distance between point of suspension and center of mass  
(it remains constant during motion)

For small angular displacement " $\theta$ " from mean position

The restoring torque is given by

$$\tau = -mg\ell \sin\theta$$

$$\tau = -mg\ell\theta \quad \because \text{for small } \theta, \sin\theta \approx \theta$$

or  $I\alpha = -mg\ell\theta$  where,  $I$  = Moment of inertia about point of suspension.

$$\text{or } \alpha = \frac{mg\ell}{I} - \theta \quad \text{or } \omega^2 = \frac{mg\ell}{I}$$

$$\text{Time period, } T = 2\pi \sqrt{\frac{I}{mg\ell}} \quad I = I_{CM} + m\ell^2$$

Where  $I_{CM}$  = moment of inertia relative to the axis which passes from the center of mass & parallel to the axis of oscillation.

$$T = 2\pi \sqrt{\frac{I_{CM} + m\ell^2}{mg\ell}}$$

where  $I_{CM} = mk^2$

$k$  = gyration radius (about axis passing from centre of mass)

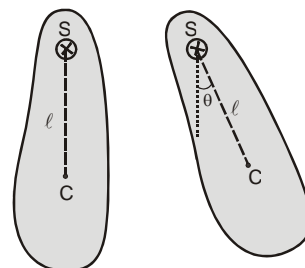
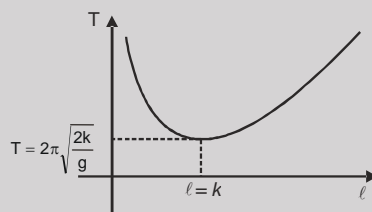
$$T = 2\pi \sqrt{\frac{mk^2 + m\ell^2}{mg\ell}} \quad T = 2\pi \sqrt{\frac{k^2 + \ell^2}{\ell g}} = 2\pi \sqrt{\frac{L_{eq}}{g}}$$

$$L_{eq} = \frac{k^2}{\ell} + \ell = \text{equivalent length of simple pendulum ;}$$

**T is minimum when  $\ell = k$ .**

$$T_{min} = 2\pi \sqrt{\frac{2k}{g}}$$

**Graph of T vs  $\ell$**



## Solved Examples

**Example 18.** A uniform rod of length 1.00 m is suspended through an end and is set into oscillation with small amplitude under gravity. Find the time period of oscillation. ( $g = 10 \text{ m/s}^2$ )

**Solution :** For small amplitude the angular motion is nearly simple harmonic and the time period is given by

$$T = 2\pi \sqrt{\frac{I}{mg(\ell/2)}} = 2\pi \sqrt{\frac{(m\ell^2/3)}{mg(\ell/2)}} = 2\pi \sqrt{\frac{2\ell}{3g}} = 2\pi \sqrt{\frac{2 \times 1.00 \text{ m}}{3 \times 10 \text{ m/s}^2}} = \frac{2\pi}{\sqrt{15}} \text{ s.}$$



## 12. TORSIONAL PENDULUM



**Resonance®**  
Educating for better tomorrow

Corp. / Reg. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)

Toll Free : 1800 258 5555 | CIN : U80302RJ2007PLC024029

ADVSH - 14



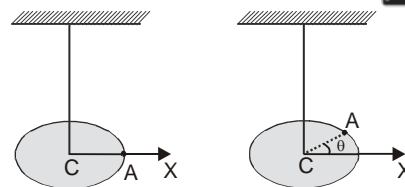
In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount, the wire applies a restoring torque causing the body to oscillate (rotate) about vertical wire, when released.

The restoring torque produced is given by

$$\tau = -C\theta \quad \text{where, } C = \text{Torsional constant}$$

or  $I\alpha = -C\theta$  where,  $I$  = Moment of inertia about the vertical axis.

$$\text{or } \alpha = -\frac{C}{I}\theta \quad \therefore \text{Time Period, } T = 2\pi\sqrt{\frac{I}{C}}$$



## Solved Examples

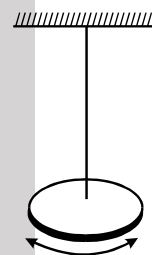
**Example 19.** A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

**Solution :** The situation is shown in figure. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\text{kg})(5.0 \times 10^{-2}\text{m})^2}{2} = 2.5 \times 10^{-4} \text{ kg} \cdot \text{m}^2.$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{or } C = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2 (2.5 \times 10^{-4} \text{ kg} \cdot \text{m}^2)}{(0.20 \text{ s})^2} = 0.25 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}.$$



## 13. SUPERPOSITION OF TWO SHM'S

### 13.1 In same direction and of same frequency.

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin (\omega t + \theta), \text{ then resultant displacement } x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin (\omega t + \theta) = A \sin (\omega t + \phi)$$

$$\text{where } A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \theta} \quad \& \quad \phi = \tan^{-1} \left[ \frac{A_2 \sin \theta}{A_1 + A_2 \cos \theta} \right]$$

If  $\theta = 0$ , both SHM's are in phase and  $A = A_1 + A_2$

If  $\theta = \pi$ , both SHM's are out of phase and  $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHM's of this case can also be found by phasor diagram also.

### 13.2 In same direction but are of different frequencies.

$$x_1 = A_1 \sin \omega_1 t$$

$$x_2 = A_2 \sin \omega_2 t$$

then resultant displacement  $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$  This resultant motion is not SHM.

### 13.3 In two perpendicular directions.

$$x = A \sin \omega t; \quad y = B \sin (\omega t + \theta)$$

**Case (i) :** If  $\theta = 0$  or  $\pi$  then  $y = \pm (B/A) x$ . So path will be straight line & resultant displacement will be

$$r = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2} \sin \omega t \quad \sqrt{A^2 + B^2}$$

which is equation of SHM having amplitude

**Case (ii) :** If  $\theta = \frac{\pi}{2}$  then.  $x = A \sin \omega t; \quad y = B \sin (\omega t + \pi/2) = B \cos \omega t$

$$\text{so, resultant will be } \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \text{ i.e. equation of an ellipse and if } A = B, \text{ then superposition will be}$$

an equation of circle. This resultant motion is not SHM.

### 13.4 Superposition of SHM's along the same direction (using phasor diagram)

If two or more SHM's are along the same line, their resultant can be obtained by vector addition by making phasor diagram.

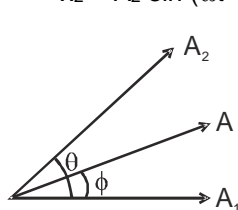




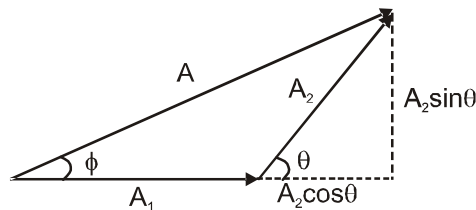


1. Amplitude of SHM is taken as length (magnitude) of vector.
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant vector gives resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

**For example :**  $x_1 = A_1 \sin \omega t$   
 $x_2 = A_2 \sin (\omega t + \theta)$



Phasor Diagram



Phasor diagram

If equation of resultant SHM is taken as  $x = A \sin (\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \theta} \Rightarrow \tan \phi = \frac{A_2 \sin \theta}{A_1 + A_2 \cos \theta}$$

### Solved Examples

**Example 20.** Find the amplitude of the simple harmonic motion obtained by combining the motions  $x_1 = (2.0 \text{ cm}) \sin \omega t$  and  $x_2 = (2.0 \text{ cm}) \sin (\omega t + \pi/3)$ .

**Solution :** The two equations given represent simple harmonic motions along X-axis with amplitudes  $A_1 = 2.0 \text{ cm}$  and  $A_2 = 2.0 \text{ cm}$ . The phase difference between the two simple harmonic motions is  $\pi/3$ . The resultant simple harmonic motion will have an amplitude  $A$  given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \delta} = \sqrt{(2.0\text{cm})^2 + (2.0\text{cm})^2 + 2(2.0\text{cm})^2 \cos \frac{\pi}{3}} = 3.5 \text{ cm}$$

**Example 21.**  $x_1 = 3 \sin \omega t$  ;  $x_2 = 4 \cos \omega t$   
 Find (i) amplitude of resultant SHM. (ii) equation of the resultant SHM.

**Solution :** First write all SHM's in terms of sine functions with positive amplitude. Keep " $\omega t$ " with positive sign.

$$\therefore x_1 = 3 \sin \omega t$$

$$x_2 = 4 \sin (\omega t + \pi/2)$$

$$A = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos \frac{\pi}{2}} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\tan \phi = \frac{4 \sin \frac{\pi}{2}}{3 + 4 \cos \frac{\pi}{2}} = \frac{4}{3}$$

$$\phi = 53^\circ$$

$$\text{equation } x = 5 \sin (\omega t + 53^\circ)$$

**Example 22**  $x_1 = 5 \sin (\omega t + 30^\circ)$

$$x_2 = 10 \cos (\omega t).$$

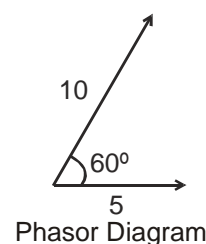
Find amplitude of resultant SHM.

**Solution :**  $x_1 = 5 \sin (\omega t + 30^\circ)$

$$x_2 = 10 \sin (\omega t + \frac{\pi}{2})$$

$$A = \sqrt{5^2 + 10^2 + 2 \times 5 \times 10 \cos 60^\circ}$$

$$= \sqrt{25 + 100 + 50} = \sqrt{175} = 5\sqrt{7}$$



Phasor Diagram

**Example 23** A particle is subjected to two simple harmonic motions  $x_1 = A_1 \sin \omega t$  and  $x_2 = A_2 \sin (\omega t + \pi/3)$ . Find (a) the displacement at  $t = 0$ , (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.



**Solution :** (a) At  $t = 0$ ,  $x_1 = A_1 \sin \omega t = 0$  and  $x_2 = A_2 \sin (\omega t + \pi/3) = A_2 \sin (\pi/3) = \frac{A_2 \sqrt{3}}{2}$ .

Thus, the resultant displacement at  $t = 0$  is  $x = x_1 + x_2 = A_2 \frac{\sqrt{3}}{2}$

(b) The resultant of the two motions is a simple harmonic motion of the same angular frequency  $\omega$ . The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\pi/3)} = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

The maximum speed is  $u_{\max} = A \omega = \omega \sqrt{A_1^2 + A_2^2 + A_1A_2}$

(c) The maximum acceleration is  $a_{\max} = A \omega^2 = \omega^2 \sqrt{A_1^2 + A_2^2 + A_1A_2}$ .

**Example 24.** A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

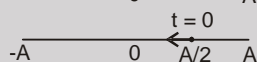
**Solution :** Let the amplitudes of the individual motions be  $A$  each. The resultant amplitude is also  $A$ . If the phase difference between the two motions is  $\delta$ ,  $A = \sqrt{A^2 + A^2 + 2A \cdot A \cos \delta}$

$$A = A \sqrt{2(1 + \cos \delta)} = 2A \cos \frac{\delta}{2} ; \quad \cos \frac{\delta}{2} = \frac{1}{2} \Rightarrow \delta = 2\pi/3.$$

## Solved Miscellaneous Problems

**Problem 1.** Write the equation of SHM for the situations shown below:

(a)  (b) 

(c) 

**Solution :**

(a) At  $t = 0$ ,  $x = +A$   
 $x = A \sin(\omega t + \phi)$  ;  $A = A \sin(\phi)$   
 $\phi = \pi/2$

$$x = A \sin\left(\omega t + \frac{\pi}{2}\right) = A \cos(\omega t)$$

(b) At  $t = 0$ ,  $x = -A$   
 $x = A \sin(\omega t + \phi)$  ;  $-A = A \sin \phi$   
 $\phi = \frac{3\pi}{2}$  ;  $x = A \sin\left(\omega t + \frac{3\pi}{2}\right)$   
 $x = -A \cos(\omega t)$

(c) At  $t = 0$ ,  $x = \frac{A}{2}$  ;  $x = A \sin(\omega t + \phi)$

$$\frac{A}{2} = A \sin(\omega t + \phi) ; \quad \frac{1}{2} = \sin \phi \Rightarrow \phi = 30^\circ, 150^\circ$$

Particle is moving towards the mean position and in negative direction.

velocity  $v = A\omega \cos(\omega t + \phi)$

At  $t = 0$ ,  $v = -ve$

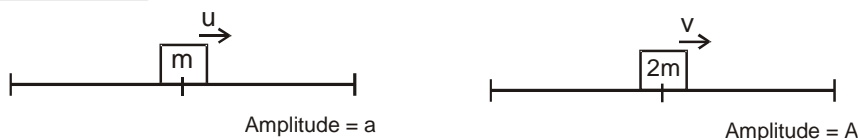
$v = A\omega \cos \phi$  hence  $\phi = 150^\circ$

$x = A \sin(\omega t + 150^\circ)$  **Ans.** (a)  $x = A \cos \omega t$  ; (b)  $x = -A \cos \omega t$  ; (c)  $x = A \sin(\omega t + 150^\circ)$

**Problem 2.** Block A of mass  $m$  is performing SHM of amplitude  $a$ . Another block B of mass  $m$  is gently placed on A when it passes through mean position and B sticks to A. Find the time period and amplitude of new SHM.



Solution :



$$\text{Time period of mass } m = 2\pi \sqrt{\frac{m}{K}}$$

$$\text{Time period of mass } 2m = 2\pi \sqrt{\frac{2m}{K}}$$

At mean position, Kinetic energy = Total Energy

$$\text{For mass } m : \frac{1}{2} mu^2 = \frac{1}{2} m \omega^2 a^2 \quad \dots(1)$$

$$\text{For mass } 2m : \frac{1}{2} 2mv^2 = \frac{1}{2} 2m A^2 \left( \frac{\omega}{\sqrt{2}} \right)^2 \quad \dots(2)$$

$$\text{By Conservation of momentum } mu = 2mv \Rightarrow v = \frac{u}{2}$$

$$\therefore \frac{1}{2} 2m \left( \frac{u}{2} \right)^2 = \frac{1}{2} 2m A^2 \left( \frac{\omega}{\sqrt{2}} \right)^2 \quad \dots(3)$$

$$\text{Divide equation (1) \& (3) ; } 4 = \frac{2a^2}{A^2}$$

$$\text{New Amplitude } A = \frac{a}{\sqrt{2}} \quad \text{Ans. } T = 2\pi \sqrt{\frac{2m}{K}} \quad \text{Amplitude} = \frac{a}{\sqrt{2}}$$

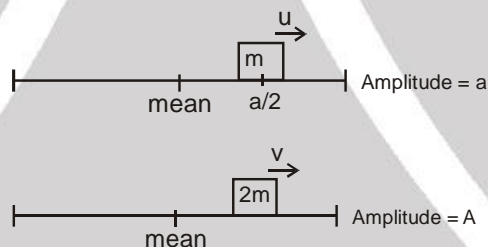
Problem 3.

Repeat the above problem assuming B is placed on A at a distance  $\frac{a}{2}$  from mean position.

Solution :

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T = 2\pi \sqrt{\frac{2m}{K}}$$



$$\text{By conservation of momentum } mu = 2mv \Rightarrow v = \frac{u}{2}$$

Kinetic Energies at  $\frac{a}{2}$ 

$$\text{For mass } m : \frac{1}{2} mu^2 = \frac{1}{2} m \omega^2 \left[ a^2 - \left( \frac{a}{2} \right)^2 \right] \quad \dots(1)$$

$$\text{For mass } 2m : \frac{1}{2} 2mv^2 = \frac{1}{2} 2m \left( \frac{\omega}{\sqrt{2}} \right)^2 \left[ A^2 - \left( \frac{a}{2} \right)^2 \right] \quad \dots(2)$$

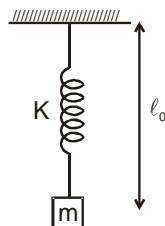
$$\frac{1}{2} 2m \frac{u^2}{4} = \frac{1}{2} 2m \frac{\omega^2}{2} \left[ A^2 - \frac{a^2}{4} \right] \quad \dots(3)$$

$$\text{Dividing equation (1) \& (3) ; } 2 = \frac{a^2 - \frac{a^2}{4}}{A^2 - \frac{a^2}{4}}$$

$$\text{New Amplitude } A = \sqrt{\frac{5}{8}} a \quad \text{Ans. } T = 2\pi \sqrt{\frac{2m}{K}}, \text{ Amplitude} = a \sqrt{\frac{5}{8}}$$



**Problem 4.** The block is allowed to fall, slowly from the position where spring is in its natural length. Find, maximum extension in the string.



**Solution :** Since the block falls slowly from rest the maximum extension occurs when  $mg = Kx_0$

$$x_0 = \frac{mg}{K} \text{ is maximum extension} \quad \text{Ans. } \frac{mg}{K}$$

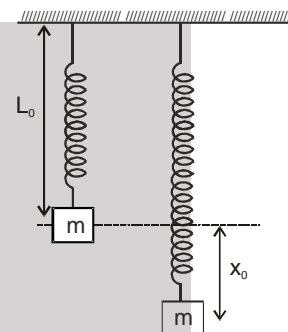
**Problem 5.** In the above problem if block is released from there, what would be maximum extension.

**Solution :** Let  $x_0$  = maximum extension

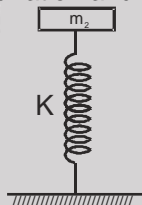
$$\text{applying conservation of energy } mgx_0 = \frac{1}{2} Kx_0^2$$

The velocity at the point of maximum extension is zero.

$$x_0 = \frac{2mg}{K} \text{ is maximum extension} \quad \text{Ans. } \frac{2mg}{K}$$



**Problem 6.** Block of mass  $m_2$  is in equilibrium as shown in figure. Another block of mass  $m_1$  is kept gently on  $m_2$ . Find the time period of oscillation and amplitude.

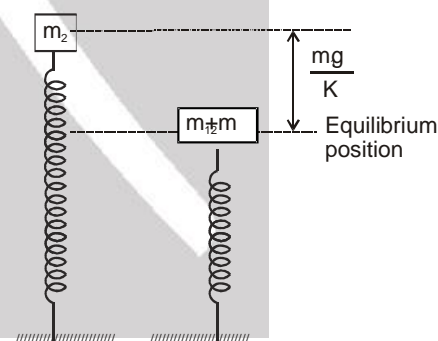


**Solution :** Time period  $T = 2\pi \sqrt{\frac{m_1 + m_2}{K}}$

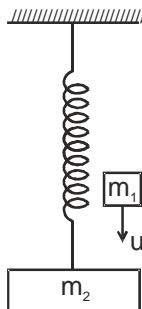
At initial position since velocity is zero it is the extreme position.

$$\text{Amplitude } A = \frac{m_1 g}{K}$$

**Ans.**  $T = 2\pi \sqrt{\frac{m_1 + m_2}{K}}$  Amplitude =  $\frac{m_1 g}{K}$



**Problem 7.** Block of mass  $m_2$  is in equilibrium and at rest. The mass  $m_1$  moving with velocity  $u$  vertically downwards collides with  $m_2$  and sticks to it. Find the energy of oscillation.





**Solution :** Let the velocity of  $m_1$  &  $m_2$  be  $v$  after collision  
By conservation of momentum  $m_1 u = (m_1 + m_2)v$

$$v = \frac{m_1 u}{m_1 + m_2}$$

$$\text{Hence, } KE = \frac{1}{2} (m_1 + m_2) v^2 = \frac{1}{2} \frac{m_1^2}{m_1 + m_2} u^2$$

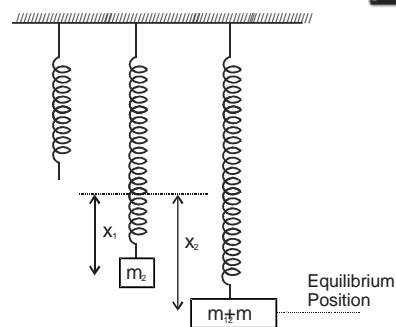
$$m_2 g = K x_1 \Rightarrow (m_1 + m_2)g = K x_2$$

$$PE = \frac{1}{2} K (x_2 - x_1)^2 = \frac{1}{2} K \left( \frac{m_1 g}{K} \right)^2 \Rightarrow PE = \frac{1}{2} \frac{m_1^2 g^2}{K}$$

Therefore energy of oscillation is  $E = KE + PE$

$$E = \frac{1}{2} \frac{m_1^2 u^2}{m_1 + m_2} + \frac{m_1^2 g^2}{2K} \Rightarrow E = \frac{1}{2} \left[ \frac{m_1^2 u^2}{m_1 + m_2} + \frac{m_1^2 g^2}{K} \right]$$

**Ans.**



**Problem 8.** A box is placed on a smooth inclined plane and it is free to move. A simple pendulum is attached in the block. Find its time period.

**Solution :**

Let  $F_p$  = Pseudo force

For equilibrium position

$$T \sin \alpha + F_p = mg \sin \theta$$

$$T \sin \alpha + ma = mg \sin \theta$$

$$\text{since } a = g \sin \theta$$

$$T \sin \alpha + mg \sin \theta = mg \sin \theta$$

$$T \sin \alpha = 0$$

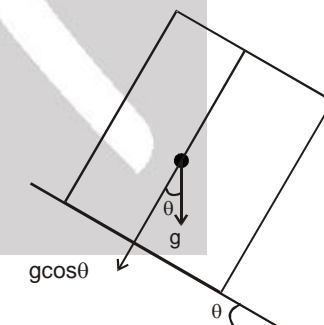
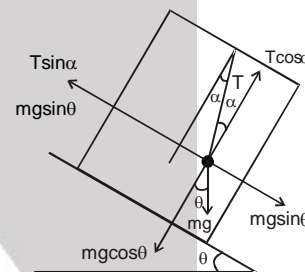
$$\therefore \alpha = 0$$

Hence in equilibrium position the string is perpendicular to incline plane.

Therefore effective 'g' is  $g_{\text{eff}} = g \cos \theta$

$$\text{Time period } T = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$$

$$T = 2\pi \sqrt{\frac{l}{g \cos \theta}}$$



**Problem 9.**  $x_1 = 5 \sin \omega t$ ;  $x_2 = 5 \sin (\omega t + 53^\circ)$ ;  $x_3 = -10 \cos \omega t$ . Find amplitude of resultant SHM

**Solution :**

$$x_1 = 5 \sin \omega t$$

$$x_2 = 5 \sin (\omega t + 53^\circ)$$

$$x_3 = -10 \cos \omega t$$

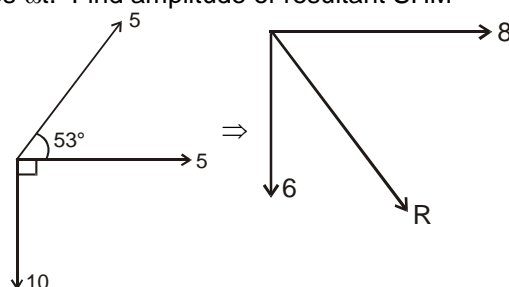
$$\text{we can write } x_3 = 10 \sin (\omega t + 270^\circ)$$

Finding the resultant amplitude by vector notation.

Resultant Amplitude

$$|R| = \sqrt{8^2 + 6^2} = 10$$

**Ans.**





## Exercise-1

Marked Questions can be used as Revision Questions.

### PART - I : SUBJECTIVE QUESTIONS

#### Section (A) : Equation of SHM

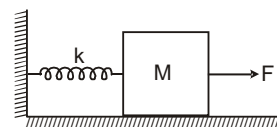
- A-1.** The equation of a particle executing SHM is  $x = (5\text{m})\sin\left[(\pi\text{s}^{-1})t + \frac{\pi}{6}\right]$ . Write down the amplitude, initial phase constant, time period and maximum speed.
- A-2.** A particle having mass 10 g oscillates according to the equation  $x = (2.0\text{ cm}) \sin [(100\text{ s}^{-1})t + \pi/6]$ . Find  
(a) the amplitude, the time period and the force constant  
(b) the position, the velocity and the acceleration at  $t = 0$ .
- A-3.** A simple harmonic motion has an amplitude  $A$  and time period  $T$ . Find the time required by it to travel directly from  
(a)  $x = 0$  to  $x = A/2$       (b)  $x = 0$  to  $x = \frac{A}{\sqrt{2}}$       (c)  $x = A$  to  $x = A/2$       (d)  $x = -\frac{A}{\sqrt{2}}$  to  $x = \frac{A}{\sqrt{2}}$   
(e)  $x = \frac{A}{\sqrt{2}}$  to  $x = A$ .
- A-4.** A particle is executing SHM with amplitude  $A$  and has maximum velocity  $v_0$ . Find its speed when it is located at distance of  $A/2$  from mean position.
- A-5.** A particle executes simple harmonic motion with an amplitude of 10 cm and time period 6 s. At  $t = 0$  it is at position  $x = 5\text{ cm}$  from mean position and going towards positive  $x$ -direction. Write the equation for the displacement  $x$  at time  $t$ . Find the magnitude of the acceleration of the particle at  $t = 4\text{ s}$ .
- A-6.** A particle is executing SHM. Find the positions of the particle where its speed is 8 cm/s, If maximum magnitudes of its velocity and acceleration are 10 cm/s and 50 cm/s<sup>2</sup> respectively.

#### Section (B) : Energy

- B-1.** A particle performing SHM with amplitude 10 cm. At What distance from mean position the kinetic energy of the particle is thrice of its potential energy?
- B-2.** An object of mass 0.2 kg executes simple harmonic oscillations along the  $x$ -axis with a frequency of  $(25/\pi)\text{ Hz}$ . At the position  $x = 0.04\text{ m}$ , the object has kinetic energy of 0.5 J and potential energy 0.4 J. Find the amplitude of oscillations [ 1994 ; 2M ]

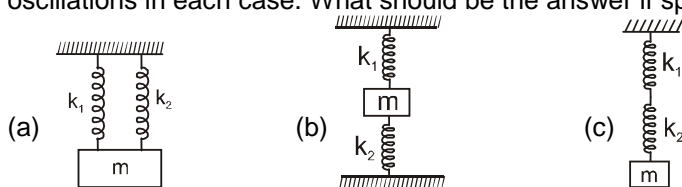
#### Section (C) : Spring mass system

- C-1.** A spring mass system has a time period of 2 second. What should be the spring constant of spring if the mass of the block is 10 grams ?
- C-2.** A body of mass 2 kg suspended through a vertical spring executes simple harmonic motion of period 4s. If the oscillations are stopped and the body hangs in equilibrium, find the potential energy stored in the spring.
- C-3.** A vertical spring-mass system with lower end of spring is fixed, made to undergo small oscillations. If the spring is stretched by 25cm, energy stored in the spring is 5J. Find the mass of the block if it makes 5 oscillations each second.
- C-4.** A spring mass system is shown in figure, spring is initially unstretched. A man starts pulling the block with constant force  $F$ . Find  
(a) The amplitude and the time period of motion of the block  
(b) The K.E. of the block at mean position  
(c) The energy stored in the spring when the block passes through the mean position

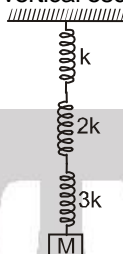




- C-5.** Three spring mass systems are shown in figure. Assuming gravity free space, find the time period of oscillations in each case. What should be the answer if space is not gravity free ?



- C-6.** Spring mass system is shown in figure. Find the elastic potential energy stored in each spring when block is at its mean position. Also find the time period of vertical oscillations. The system is in vertical plane.



### Section (D) : Simple Pendulum

- D-1.** Find the length of seconds pendulum at a place where  $g = \pi^2 \text{ m/s}^2$ .
- D-2.** Instantaneous angle (in radian) between string of a simple pendulum and vertical is given by  $\theta = \frac{\pi}{180} \sin 2\pi t$ . Find the length of the pendulum if  $g = \pi^2 \text{ m/s}^2$ .
- D-3.** A pendulum clock is accurate at a place where  $g = 9.8 \text{ m/s}^2$ . Find the value of  $g$  at another place where clock becomes slow by 24 seconds in a day (24 hrs).
- D-4.** A pendulum is suspended in a lift and its period of oscillation is  $T_0$  when the lift is stationary.
- What will be the period  $T$  of oscillation of pendulum, if the lift begins to accelerate downwards with an acceleration equal to  $\frac{3g}{4}$ ?
  - What must be the acceleration of the lift for the period of oscillation of the pendulum to be  $\frac{T_0}{2}$ ?

### Section (E) : Compound Pendulum & Torsional Pendulum

- E-1.** Compound pendulums are made of :
- A rod of length  $\ell$  suspended through a point located at distance  $\ell/4$  from centre of rod.
  - A ring of mass  $m$  and radius  $r$  suspended through a point on its periphery.
  - A uniform square plate of edge  $a$  suspended through a corner.
  - A uniform disc of mass  $m$  and radius  $r$  suspended through a point  $r/2$  away from the centre.
- Find the time period in each case.
- E-2.** Two compound pendulums are made of :
- A disc of radius  $r$  and
  - A uniform rod of length  $L$ . Find the minimum possible time period and distance between centre and point of suspension in each case.

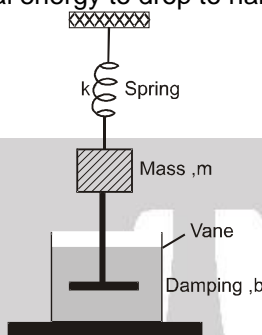
### Section (F) : Superposition of SHM

- F-1.** A particle is subjected to two SHM's simultaneously  $X_1 = a_1 \sin \omega t$  and  $X_2 = a_2 \sin(\omega t + \phi)$  Where  $a_1 = 3.0 \text{ cm}$ ,  $a_2 = 4.0 \text{ cm}$ . Find resultant amplitude if the phase difference  $\phi$  has values (a)  $0^\circ$  (b)  $60^\circ$  (c)  $90^\circ$ .
- F-2.** A particle is subjected to three SHM's in same direction simultaneously each having equal amplitude  $a$  and equal time period. The phase of the second motion is  $30^\circ$  ahead of the first and the phase of the third motion is  $30^\circ$  ahead of the second. Find the amplitude of the resultant motion.
- F-3.** A particle simultaneously participates in two mutually perpendicular oscillations  $x = \sin \pi t$  &  $y = 2 \cos 2 \pi t$ . Write the equation of trajectory of the particle.



**Section (G) : For JEE-Main**

- G-1.** In forced oscillation of a particle, the amplitude is maximum for a frequency  $\omega_1$  of the force, while the energy is maximum for a frequency  $\omega_2$  of the force. What is the relation between  $\omega_1$  and  $\omega_2$  ?
- G-2.** For the damped oscillator shown in Figure, the mass of the block is 200 g,  $k = 80 \text{ Nm}^{-1}$  and the damping constant  $b$  is  $40 \text{ gs}^{-1}$ . Calculate
- The period of oscillation,
  - Time taken for its amplitude of vibrations to drop to half of its initial value
  - The time for the mechanical energy to drop to half initial value.

**PART - II : ONLY ONE OPTION CORRECT TYPE****Section (A) : Equation of SHM**

- A-1.** According to a scientists, he applied a force  $F = -cx^{1/3}$  on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether  $c$  is a constant or not. Assume that he had worked only with positive  $x$  then :
- as  $x$  increases  $c$  also increases
  - as  $x$  increases  $c$  decreases
  - as  $x$  increases  $c$  remains constant
  - the motion cannot be SHM
- A-2.** A particle performing SHM takes time equal to  $T$  (time period of SHM) in consecutive appearances at a particular point. This point is :
- An extreme position
  - The mean position
  - Between positive extreme and mean position
  - Between negative extreme and mean position
- A-3.** A particle executing linear SHM. Its time period is equal to the smallest time interval in which particle acquires a particular velocity  $\vec{v}$ , the magnitude of  $\vec{v}$  may be :
- Zero
  - $V_{\max}$
  - $\frac{V_{\max}}{2}$
  - $\frac{V_{\max}}{\sqrt{2}}$
- A-4.** If  $\vec{F}$  is force vector,  $\vec{v}$  is velocity vector,  $\vec{a}$  vector is acceleration vector and  $\vec{r}$  vector is displacement vector with respect to mean position than which of the following quantities are always non-negative in a simple harmonic motion along a straight line?
- $\vec{F} \cdot \vec{a}$
  - $\vec{v} \cdot \vec{r}$
  - $\vec{a} \cdot \vec{r}$
  - $\vec{F} \cdot \vec{r}$
- A-5.** Two SHM's are represented by  $y = a \sin (\omega t - \phi)$  and  $y = b \cos (\omega t - \phi)$ . The phase difference between the two is :
- $\frac{\pi}{2}$
  - $\frac{\pi}{4}$
  - $\frac{\pi}{6}$
  - $\frac{3\pi}{4}$
- A-6.** How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.
- 12s
  - 2s
  - 4s
  - 6s
- A-7.** The magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is
- $\frac{2A\omega^2}{\pi}$
  - $\frac{A\omega^2}{2\pi}$
  - $\frac{A\omega^2}{\sqrt{2}\pi}$
  - Zero



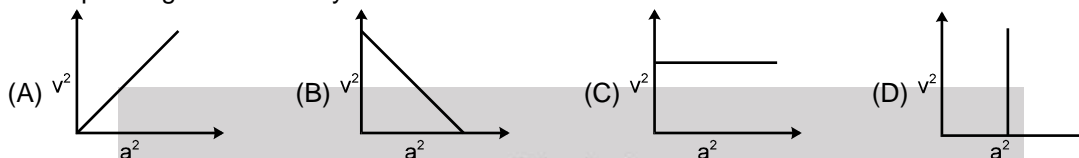


**A-8.** A particle performing SHM on the y axis according to equation  $y = A + B \sin \omega t$ . Its amplitude is :

- (A) A (B) B (C)  $A + B$  (D)  $\sqrt{A^2 + B^2}$

**A-9.** Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement from mean position is half of their amplitudes. The phase-difference between them is  
(A)  $0^\circ$  (B)  $120^\circ$  (C)  $180^\circ$  (D)  $135^\circ$

**A-10.** A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is



### Section (B) : Energy

**B-1.** A body executing SHM passes through its equilibrium. At this instant, it has  
(A) maximum potential energy (B) maximum kinetic energy  
(C) minimum kinetic energy (D) maximum acceleration

**B-2.** The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is  
(A)  $\sqrt{2}A$  (B)  $\frac{A}{2}$  (C)  $\frac{A}{\sqrt{2}}$  (D)  $\frac{\sqrt{2}}{3}A$

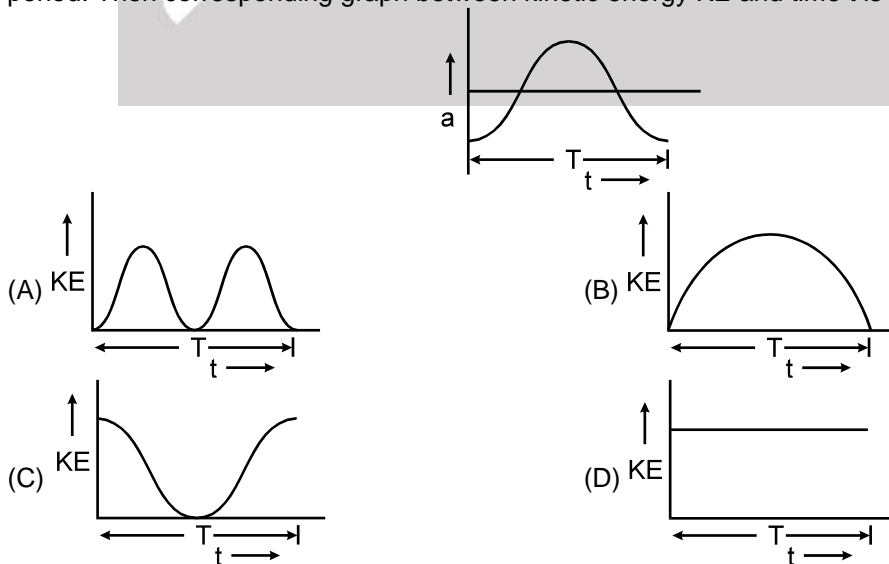
**B-3.** A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is  $8 \times 10^{-3} \text{ J}$ . The equation of motion of this particle when the initial phase of oscillation is  $45^\circ$  can be given by [I.I.T. 1991]

- (A)  $0.1 \cos\left(4t + \frac{\pi}{4}\right)$  (B)  $0.1 \sin\left(4t + \frac{\pi}{4}\right)$  (C)  $0.4 \sin\left(t + \frac{\pi}{4}\right)$  (D)  $0.2 \sin\left(\frac{\pi}{2} + 2t\right)$

**B-4.** For a particle performing SHM :

- (A) The kinetic energy is never equal to the potential energy  
(B) the kinetic energy is always equal to the potential energy  
(C) The average kinetic energy in one time period is equal to the average potential in this period  
(D) The average kinetic energy in any time interval is equal to average potential energy in that interval

**B-5.** Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time t is correctly represented by





- B-6.** A particle performs S.H.M. of amplitude  $A$  along a straight line. When it is at a distance  $\frac{\sqrt{3}}{2}A$  from mean position, its kinetic energy gets increased by an amount  $\frac{1}{2}m\omega^2 A^2$  due to an impulsive force.

Then its new amplitude becomes:

- (A)  $\frac{\sqrt{5}}{2}A$  (B)  $\frac{\sqrt{3}}{2}A$  (C)  $\sqrt{2}A$  (D)  $\sqrt{5}A$

### Section (C) : Spring mass system

- C-1.** Two spring mass systems have equal mass and spring constant  $k_1$  and  $k_2$ . If the maximum velocities in two systems are equal then ratio of amplitude of 1st to that of 2nd is :

- (A)  $\sqrt{k_1/k_2}$  (B)  $k_1/k_2$  (C)  $k_2/k_1$  (D)  $\sqrt{k_2/k_1}$

- C-2.** A toy car of mass  $m$  is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is  $k$  and surface is frictionless. The car is displaced from mean position by  $x$  cm and released. At the mean position the ribbons are undeformed. Vibration period is



- (A)  $2\pi\sqrt{\frac{m(2k)}{k^2}}$  (B)  $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$  (C)  $2\pi\sqrt{\frac{m}{k}}$  (D)  $2\pi\sqrt{\frac{m}{k+k}}$

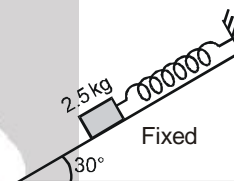
- C-3.** A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half : [REE - 1988]

- (A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

- C-4.** A ball of mass  $m$  kg hangs from a spring of spring constant  $k$ . The ball oscillates with a period of  $T$  seconds. If the ball is removed, the spring is shortened (with respect to length in mean position) by

- (A)  $\frac{gT^2}{(2\pi)^2}$  metre (B)  $\frac{3T^2g}{(2\pi)^2}$  metre (C)  $\frac{Tm}{k}$  metre (D)  $\frac{Tk}{m}$  metre

- C-5.** A smooth inclined plane having angle of inclination  $30^\circ$  with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is

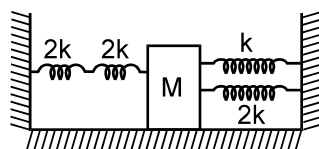


- (A) 0.707 (B) 7.07 (C) 1.414 (D) 14.14

- C-6.** A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is  $T$ . If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

- (A) remain  $T$  (B) become  $2T$  (C) become  $T/2$  (D) become  $T/\sqrt{2}$

- C-7.** Four massless springs whose force constants are  $2k$ ,  $2k$ ,  $k$  and  $2k$  respectively are attached to a mass  $M$  kept on a frictionless plane (as shown in figure). If the mass  $M$  is displaced in the horizontal direction, then the frequency of the system. [JEE 1990]



- (A)  $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$  (B)  $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$  (C)  $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$  (D)  $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

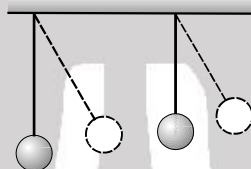




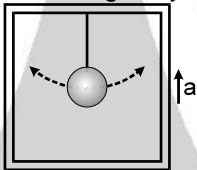
- C-8.** The total mechanical energy of a particle of mass  $m$  executing SHM with the help of a spring is  $E = (1/2)m\omega^2 A^2$ . If the particle is replaced by another particle of mass  $m/2$  while the amplitude  $A$  remains same. New mechanical energy will be :
- (A)  $\sqrt{2} E$  (B)  $2E$  (C)  $E/2$  (D)  $E$

### Section (D) : Simple Pendulum

- D-1.** Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.
- (A)  $\frac{49}{81}$  (B)  $\frac{7}{9}$  (C)  $\frac{50}{81}$  (D)  $\frac{1}{2}$
- D-2.** Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) :



- (A) 1 vibration (B) 3 vibrations (C) 4 vibrations (D) 5 vibrations
- D-3.** A scientist measures the time period of a simple pendulum as  $T$  in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



- (A)  $\frac{T}{4}$  (B)  $4T$  (C)  $\frac{2}{\sqrt{5}} T$  (D)  $\frac{\sqrt{5}}{2} T$
- D-4.** A simple pendulum has some time period  $T$ . What will be the percentage change in its time period if its amplitude is decreased by 5%?
- (A) 6 % (B) 3 % (C) 1.5 % (D) 0 %
- D-5.** A simple pendulum with length  $\ell$  and bob of mass  $m$  executes SHM of small amplitude  $A$ . The maximum tension in the string will be
- (A)  $mg(1 + A/\ell)$  (B)  $mg(1 + A/\ell)^2$  (C)  $mg[1 + (A/\ell)^2]$  (D)  $2 mg$

### Section (E) : Compound Pendulum & Torsional Pendulum

- E-1.** A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then the orientation is maintained. If the sphere is then released, its time period of the oscillation will be :
- (A)  $\pi$  second (B)  $\sqrt{2} \pi$  second (C)  $2\pi$  second (D)  $4\pi$  second
- E-2.** A metre stick swinging about its one end oscillates with frequency  $f_0$ . If the bottom half of the stick was cut off, then its new oscillation frequency will be :
- (A)  $f_0$  (B)  $\sqrt{2} f_0$  (C)  $2f_0$  (D)  $2 f_0$

### Section (F) : Superposition of SHM

- F-1.** When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed
- (A) the resulting motion is uniform circular motion.
- (B) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.
- (C) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.
- (D) the two S.H.M. will cancel each other.





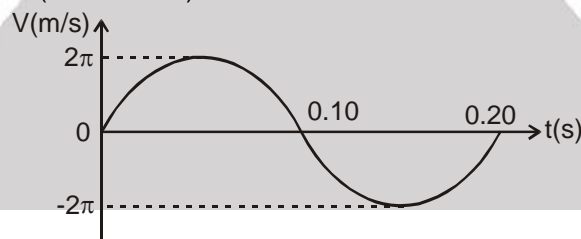
- F-2.** The position of a particle in motion is given by  $y = C\sin\omega t + D\cos\omega t$  w.r.t. origin. Then motion of the particle is:
- (A) SHM with amplitude  $C+D$  (B) SHM with amplitude  $\sqrt{C^2 + D^2}$   
 (C) SHM with amplitude  $\frac{(C+D)}{2}$  (D) not SHM
- F-3.** A simple harmonic motion is given by  $y = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$ . What is the amplitude of motion if  $y$  is in m ?  
 (A) 100 cm (B) 5 m (C) 200 cm (D) 1000 cm
- F-4.** The position vector of a particle moving in x-y plane is given by  $\vec{r} = (A \sin\omega t) \hat{i} + (A \cos\omega t) \hat{j}$  then motion of the particle is :  
 (A) SHM (B) on a circle (C) on a straight line (D) with constant acceleration

### Section (G) : For JEE Main

- G-1.** When an oscillator completes 100 oscillations its amplitude reduced to  $\frac{1}{3}$  of initial value. What will be its amplitude, when it completes 200 oscillations : [AIPMT 2002]  
 (A)  $\frac{1}{8}$  (B)  $\frac{2}{3}$  (C)  $\frac{1}{6}$  (D)  $\frac{1}{9}$
- G-2.** The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are :  
 (A)  $\text{kgms}^{-1}$  (B)  $\text{kgms}^{-2}$  (C)  $\text{kgs}^{-1}$  (D) kgs

## PART - III : MATCH THE COLUMN

- 1.** A simple harmonic oscillator consists of a block attached to a spring with  $k = 200 \text{ N/m}$ . The block slides on a frictionless horizontal surface, with equilibrium point  $x = 0$ . A graph of the block's velocity  $v$  as a function of time  $t$  is shown. Correctly match the required information in the left column with the values given in the right column. (Use  $\pi^2 = 10$ )



### Left Column

- (A) The block's mass in kg  
 (B) The block's displacement at  $t = 0$  in metres  
 (C) The block's acceleration at  $t = 0.10 \text{ s}$  in  $\text{m/s}^2$   
 (D) The block's maximum kinetic energy in Joule

### Right Column

- (p)  $-0.20$   
 (q)  $-200$   
 (r)  $0.20$   
 (s)  $4.0$





2. In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

**Column-I**

- (A) A simple pendulum of length ' $\ell$ ' oscillating with small amplitude in a lift moving down with retardation  $g/2$ .
- (B) A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a stationary lift, stretches the spring by length ' $\ell$ ' in equilibrium. It's time period when lift moves up with an acceleration  $g/2$  is
- (C) The time period of small oscillation of a uniform rod of length ' $\ell$ ' smoothly hinged at one end. The rod oscillates in vertical plane.
- (D) A cubical block of edge ' $\ell$ ' and specific gravity  $1/2$  is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is

**Column-II**

(p)  $T = 2\pi \sqrt{\frac{2\ell}{3g}}$

(q)  $T = 2\pi \sqrt{\frac{\ell}{g}}$

(r)  $T = 2\pi \sqrt{\frac{2\ell}{g}}$

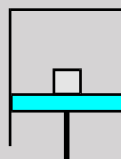
(s)  $T = 2\pi \sqrt{\frac{\ell}{2g}}$

## Exercise-2

Marked Questions can be used as Revision Questions.

### PART - I : ONLY ONE OPTION CORRECT TYPE

1. A block of mass  $m$  is resting on a piston as shown in figure which is moving vertically with a SHM of period  $1$  s. The minimum amplitude of motion at which the block and piston separate is :

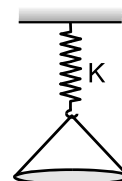


- (A)  $0.25$  m      (B)  $0.52$  m      (C)  $2.5$  m      (D)  $0.15$  m

2. The potential energy of a particle of mass ' $m$ ' situated in a unidimensional potential field varies as  $U(x) = U_0 [1 - \cos ax]$ , where  $U_0$  and  $a$  are constants. The time period of small oscillations of the particle about the mean position :

- (A)  $2\pi \sqrt{\frac{m}{aU_0}}$       (B)  $2\pi \sqrt{\frac{am}{U_0}}$       (C)  $2\pi \sqrt{\frac{m}{a^2U_0}}$       (D)  $2\pi \sqrt{\frac{a^2m}{U_0}}$

3. A solid ball of mass  $m$  is made to fall from a height  $H$  on a pan suspended through a spring of spring constant  $K$  as shown in figure. If the ball does not rebound and the pan is massless, then amplitude of oscillation is

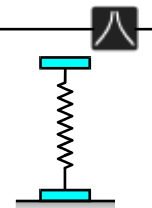


- (A)  $\frac{mg}{K}$       (B)  $\frac{mg}{K} \left( 1 + \frac{2HK}{mg} \right)^{1/2}$
- (C)  $\frac{mg}{K} + \left( \frac{2HK}{mg} \right)^{1/2}$       (D)  $\frac{mg}{K} \left[ 1 + \left( 1 + \frac{2HK}{mg} \right)^{1/2} \right]$





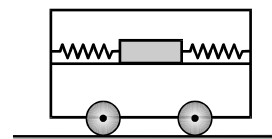
## Simple Harmonic Motion



4. Two plates of same mass are attached rigidly to the two ends of a spring as shown in figure. One of the plates rests on a horizontal surface and the other results a compression  $y$  of the spring when it is in equilibrium state. The further minimum compression required, so that after the force causing compression is removed the lower plate is lifted off the surface, will be :

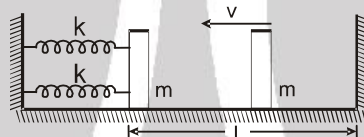
(A)  $0.5y$  (B)  $3y$  (C)  $2y$  (D)  $y$

5. Two springs, each of spring constant  $k$ , are attached to a block of mass  $m$  as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass  $M$ . If the block is displaced by  $x$  cm and released, the period of oscillation is :



(A)  $T = 2\pi \sqrt{\frac{Mm}{2k}}$  (B)  $T = 2\pi \sqrt{\frac{(M+m)}{kmM}}$  (C)  $T = 2\pi \sqrt{\frac{mM}{2k(M+m)}}$  (D)  $T = 2\pi \frac{(M+m)^2}{k}$

6. The right block in figure moves at a speed  $V$  towards the left block placed in equilibrium. All the surfaces are smooth and all the collisions are elastic. Find the time period of periodic motion. Neglect the width of the blocks.



(A)  $\pi \sqrt{\frac{m}{2k}} + \frac{2L}{v}$  (B)  $\pi \sqrt{\frac{m}{2k}} + \frac{L}{v}$  (C)  $\pi \sqrt{\frac{m}{2k}} - \frac{L}{v}$  (D)  $\pi \sqrt{\frac{m}{k}} + \frac{L}{v}$

7. The bob in a simple pendulum of length  $\ell$  is released at  $t = 0$  from the position of small angular displacement  $\theta_0$ . Linear displacement of the bob at any time  $t$  from the mean position is given by

(A)  $\ell \theta_0 \cos \sqrt{\frac{g}{\ell}} t$  (B)  $\ell \sqrt{\frac{g}{\ell}} t \cos \theta_0$  (C)  $\ell g \sin \theta_0$  (D)  $\ell \theta_0 \sin \sqrt{\frac{g}{\ell}} t$

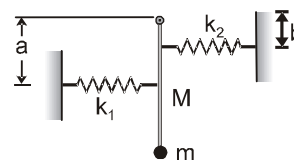
8. The period of small oscillations of a simple pendulum of length  $\ell$  if its point of suspension  $O$  moves with a constant acceleration  $\alpha = \alpha_1 \hat{i} - \alpha_2 \hat{j}$  with respect to earth is ( $\hat{i}$  and  $\hat{j}$  are unit vectors in horizontal and vertically upward directions respectively)

(A)  $T = 2\pi \sqrt{\frac{\ell}{\{(g - \alpha_2)^2 + \alpha_1^2\}^{1/2}}}$  (B)  $T = 2\pi \sqrt{\frac{\ell}{\{(g - \alpha_1)^2 + \alpha_2^2\}^{1/2}}}$   
(C)  $T = 2\pi \sqrt{\frac{\ell}{g}}$  (D)  $T = 2\pi \sqrt{\frac{\ell}{(g^2 + \alpha_1^2)^{1/2}}}$

9. A simple pendulum ; a physical pendulum; a torsional pendulum and a spring-mass system, each of same frequency are taken to the Moon. If frequencies are measured on the moon, which system or systems will have it unchanged ?

(A) spring-mass system and torsional pendulum. (B) only spring-mass system.  
(C) spring-mass system and physical pendulum. (D) None of these

10. A rod of mass  $M$  and length  $L$  is hinged at its one end and carries a particle of mass  $m$  at its lower end. A spring of force constant  $k_1$  is installed at distance  $a$  from the hinge and another of force constant  $k_2$  at a distance  $b$  as shown in the figure. If the whole arrangement rests on a smooth horizontal table top, the frequency of vibration is



(A)  $\frac{1}{2\pi} \sqrt{\frac{k_1 a^2 + k_2 b^2}{L^2(m + \frac{M}{3})}}$  (B)  $\frac{1}{2\pi} \sqrt{\frac{k_2 + k_1}{M + m}}$  (C)  $\frac{1}{2\pi} \sqrt{\frac{k_2 + k_1 \frac{a^2}{b^2}}{\frac{4M}{3} + m}}$  (D)  $\frac{1}{2\pi} \sqrt{\frac{k_1 + \frac{k_2 b^2}{a^2}}{\frac{4}{3}m + M}}$







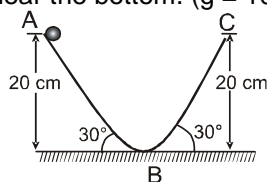
11. A particle moves along the X-axis according to the equation  $x = 10 \sin^3(\pi t)$ . The amplitudes and frequencies of component SHMs are  
 (A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2 (B) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2  
 (C) amplitude 10, 10 ; frequencies 1/2, 1/2 (D) amplitude 30/4, 10 ; frequencies 3/2, 2
12. The amplitude of a particle due to superposition of following S.H.Ms. Along the same line is  
 $X_1 = 2 \sin 50\pi t$  ;  $X_2 = 10 \sin(50\pi t + 37^\circ)$   
 $X_3 = -4 \sin 50\pi t$  ;  $X_4 = -12 \cos 50\pi t$   
 (A)  $4\sqrt{2}$  (B) 4 (C)  $6\sqrt{2}$  (D) none of these
13. When a body is suspended from a fixed point by a spring, the angular frequency of its vertical oscillations is  $\omega_1$ . When a different spring is used, the angular frequency is  $\omega_2$ . The angular frequency of vertical oscillations when both the springs are used together in series is given by [Olympiad (Stage-1) 2017]  
 (A)  $\omega = \left[ \omega_1^2 + \omega_2^2 \right]^{\frac{1}{2}}$  (B)  $\omega = \left[ \frac{\omega_1^2 + \omega_2^2}{2} \right]^{\frac{1}{2}}$  (C)  $\omega = \left[ \frac{\omega_1^2 \omega_2^2}{(\omega_1^2 + \omega_2^2)} \right]^{\frac{1}{2}}$  (D)  $\omega = \left[ \frac{\omega_1^2 \omega_2^2}{2(\omega_1^2 + \omega_2^2)} \right]^{\frac{1}{2}}$
14. A particle performs simple harmonic motion at a frequency  $f$ . The frequency at which its kinetic energy varies is : [Olympiad (Stage-1) 2017]  
 (A)  $f$  (B)  $2f$  (C)  $4f$  (D)  $f/2$
15. A particle rests in equilibrium under two forces of repulsion whose centres are at distance of  $a$  and  $b$  from the particle. The forces vary as the cube of the distance. The forces per unit mass are  $k$  and  $k'$  respectively. If the particle be slightly displaced towards one of them the motion is simple harmonic with the time period equal to [Olympiad (Stage-1) 2017]  
 (A)  $\frac{2\pi}{\sqrt{3\left(\frac{k}{a^3} + \frac{k'}{b^3}\right)}}$  (B)  $\frac{2\pi}{\sqrt{\left(\frac{k}{a^3} + \frac{k'}{b^3}\right)}}$  (C)  $\frac{2\pi}{\sqrt{\left(\frac{k}{a^4} + \frac{k'}{b^4}\right)}}$  (D)  $\frac{2\pi}{\sqrt{3\left(\frac{k}{a} + \frac{k'}{b}\right)}}$

## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

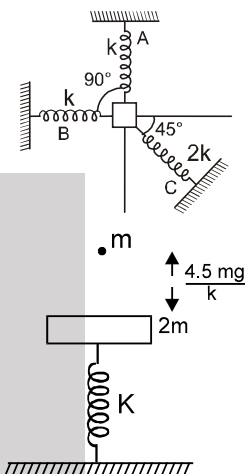
1. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the co-ordinates (3, 0) and (0, 4) respectively. At  $t = 0$ , B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. If the maximum and minimum distances between A and B is  $s_1$  and  $s_2$  then find  $s_1 + s_2$  (in cm).
2. Two particles P and Q describe S.H.M. of same amplitude  $a$ , same frequency  $f$  along the same straight line from the same mean position. The maximum distance between the two particles is  $a\sqrt{2}$ . If the initial phase difference between the particles is  $\pi/N$  then find  $N$ .
3. A street car moves rectilinearly from station A (here car stops) to the next station B (here also car stops) with an acceleration varying according to the law  $f = a - bx$ , where  $a$  and  $b$  are positive constants and  $x$  is the distance from station A. If the maximum distance between the two stations is  $x = \frac{Na}{b}$  then find  $N$ .
4. A particle is oscillating in a straight line about a centre O, with a force directed towards O. When at a distance ' $x$ ' from O, the force is  $mn^2x$  where ' $m$ ' is the mass and ' $n$ ' is a constant. The amplitude is  $a = 15$  cm. When at a distance  $\sqrt{3}\frac{a}{2}$  from O the particle receives a blow in the direction of motion which generates an extra velocity  $na$ . If the velocity is away from O at the time of blow and the new amplitude becomes  $k\sqrt{3}$  cm, then find  $k$ .
5. Two particles  $P_1$  and  $P_2$  are performing SHM along the same line about the same mean position. Initially they are at their positive extreme positions. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time after which the separation between the particles become 6 cm.



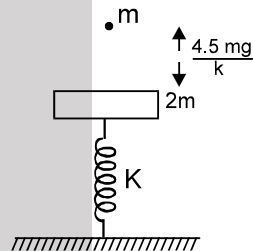
6. Assuming all the surfaces to be smooth, if the time period of motion of the ball is  $N \times 10^{-1}$  sec then find N. Neglect the small effect of bend near the bottom. ( $g = 10 \text{ m/s}^2$ )



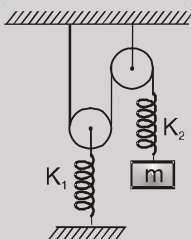
7. A block of mass  $m$  is attached to three springs A, B and C having force constants  $k$ ,  $k$  and  $2k$  respectively as shown in figure. If the block is slightly pushed against spring C. If the angular frequency of oscillations is  $\sqrt{\frac{Nk}{m}}$ , then find N. The system is placed on horizontal smooth surface.



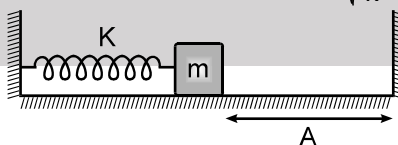
8. In the figure shown mass  $2m$  is at rest and in equilibrium. A particle of mass  $m$  is released from height  $\frac{4.5mg}{k}$  from plate. The particle sticks to the plate. Neglecting the duration of collision. Starting from the time when the particle sticks to plate to the time when the spring is in maximum compression for the first time is  $2\pi\sqrt{\frac{m}{ak}}$  then find  $a$ .



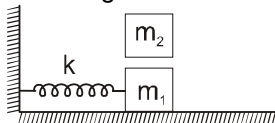
9. For given spring mass system, If the time period of small oscillations of block about its mean position is  $\pi\sqrt{\frac{nm}{K}}$ , then find  $n$ . Assume ideal conditions. The system is in vertical plane and take  $K_1 = 2K$ ,  $K_2 = K$ .



10. In the figure shown the spring is relaxed and mass  $m$  is attached to the spring. The spring is compressed by  $2A$  and released at  $t = 0$ . Mass  $m$  collides with the wall and loses two third of its kinetic energy and returns. Starting from  $t = 0$ , find the time taken by it to come back to rest again (instant at which spring is again under maximum compression). Take  $\sqrt{\frac{m}{k}} = \frac{12}{\pi}$



11. A block of mass  $4 \text{ kg}$  attached with spring of spring constant  $100 \text{ N/m}$  is executing SHM of amplitude  $0.1 \text{ m}$  on smooth horizontal surface as shown in figure. If another block of mass  $5 \text{ kg}$  is gently placed on it, at the instant it passes through the mean position and new amplitude of motion is  $n^{-1}$  meter then find  $n$ . Assuming that two blocks always move together.



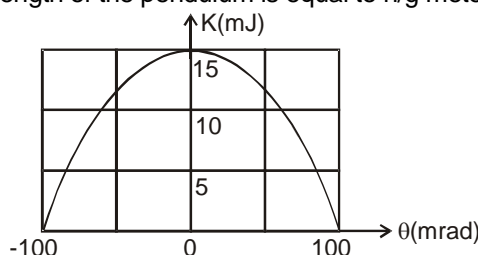
12. The period of oscillation of a simple pendulum of length  $L$  suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination  $\alpha = 60^\circ$  is given by  $\pi\sqrt{\frac{XL}{g}}$  then find  $X$ .

[I.I.T. (Scr.) 2000, 1/35]





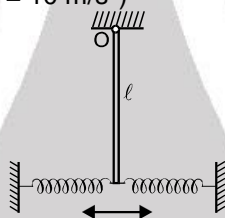
13. Figure shows the kinetic energy  $K$  of a simple pendulum versus its angle  $\theta$  from the vertical. The pendulum bob has mass  $0.2 \text{ kg}$ . If the length of the pendulum is equal to  $n/g$  meter, then find  $n$  ( $g = 10 \text{ m/s}^2$ ).



14. The bob of a simple pendulum executes simple harmonic motion in water with a period ' $t$ ', while the period of oscillation of the bob is  $t_0$  in air. Neglecting frictional force of water and given that the density of the bob is  $(4/3) \times 1000 \text{ kg/m}^3$ . Find  $\frac{t}{t_0}$ . [AIEEE 2004]

15. A solid sphere of radius  $R$  is half submerged in a liquid of density  $\rho$ . The sphere is slightly pushed down and released, If the frequency of small oscillations is  $\sqrt{\frac{3}{nR}}$ , then find  $n$ . Take  $\pi = \sqrt{g}$ . [JEE (Mains) 2004, 2/60]

16. If the angular frequency of small oscillations of a thin uniform vertical rod of mass  $m$  and length  $\ell$  hinged at the point  $O$  (Fig.) is  $\sqrt{\frac{n}{\ell}}$ , then find  $n$ . The force constant for each spring is  $K/2$  and take  $K = \frac{2mg}{\ell}$ . The springs are of negligible mass. ( $g = 10 \text{ m/s}^2$ )



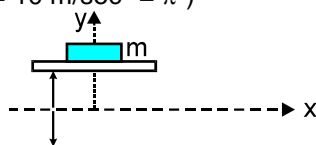
### PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A particle moves on the  $X$ -axis according to the equation  $x = x_0 \sin^2 \omega t$ . The motion is simple harmonic  
(A) with amplitude  $x_0/2$  (B) with amplitude  $2x_0$  (C) with time period  $\frac{2\pi}{\omega}$  (D) with time period  $\frac{\pi}{\omega}$
2. Which of the following functions represent SHM?  
(A)  $\sin 2\omega t$  (B)  $\sin^2 \omega t$  (C)  $\sin \omega t + 2 \cos \omega t$  (D)  $\sin \omega t + \cos 2\omega t$
3. The speed  $v$  of a particle moving along a straight line, when it is at a distance ( $x$ ) from a fixed point of the line is given by  $v^2 = 108 - 9x^2$  (assuming mean position to have zero phase constant) (all quantities are in cgs units) :  
(A) the motion is uniformly accelerated along the straight line  
(B) the magnitude of the acceleration at a distance  $3 \text{ cm}$  from the fixed point is  $27 \text{ cm/s}^2$   
(C) the motion is simple harmonic about the given fixed point.  
(D) the maximum displacement from the fixed point is  $4 \text{ cm}$ .
4. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of  $40 \text{ cm}$ . The block just loses contact with the plank when the latter is at momentary rest. Then :  
(A) the period of oscillation is  $\left(\frac{2\pi}{5}\right)$  seconds  
(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.  
(C) the block weighs  $0.5$  times its weight on the plank halfway up  
(D) the block weighs  $1.5$  times its weight on the plank halfway down  
(E) the block weighs its true weight on the plank when the latter moves fastest

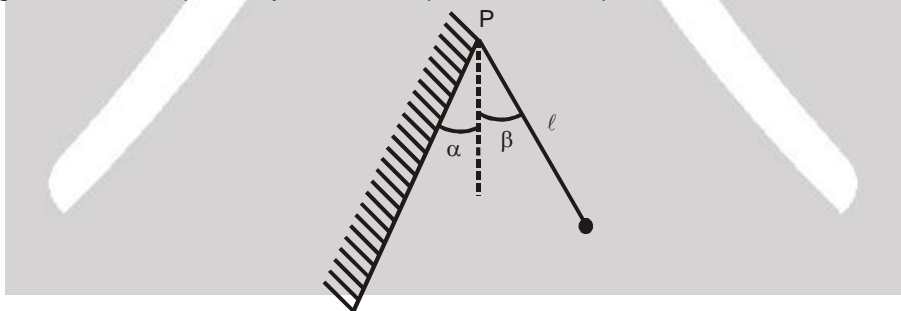




5. As shown in figure a horizontal platform with a mass  $m$  placed on it is executing SHM along  $y$ -axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is : ( $g = 10 \text{ m/sec}^2 = \pi^2$ )



- (A)  $\frac{10}{\pi} \text{ s}$  (B)  $\frac{\pi}{10} \text{ s}$  (C)  $\frac{\pi}{\sqrt{10}} \text{ s}$  (D)  $\frac{1}{\sqrt{10}} \text{ s}$ .
6. For a body executing SHM with amplitude  $A$ , time period  $T$ , maximum velocity  $v_{\max}$  and phase constant zero, which of the following statements are correct for  $0 \leq t \leq \frac{T}{4}$  ( $y$  is displacement from mean position) ?
- (A) At  $y = (A/2)$ ,  $v > (v_{\max}/2)$  (B) for  $v = (v_{\max}/2)$ ,  $y > (A/2)$   
 (C) For  $t = (T/8)$ ,  $y > (A/2)$  (D) For  $y = (A/2)$ ,  $t < (T/8)$
7. The potential energy of a particle of mass 0.1 kg, moving along the  $x$ -axis, is given by  $U = 5x(x - 4) \text{ J}$ , where  $x$  is in meters. It can be concluded that
- (A) the particle is acted upon by a constant force (B) the speed of the particle is maximum at  $x = 2 \text{ m}$   
 (C) the particle executes SHM (D) the period of oscillation of the particle is  $(\pi/5) \text{ sec}$
8. A particle free to move along the  $x$ -axis has potential energy given by  $U(x) = k[1 - e^{-x^2}]$  for  $-\infty \leq x \leq +\infty$ , where  $k$  is a positive constant of appropriate dimensions. Then select the incorrect options : [JEE - 1999, 2/200]
- (A) at points away from the origin, the particle is in unstable equilibrium.  
 (B) for any finite non-zero value of  $x$ , there is a force directed away from the origin.  
 (C) if its total mechanical energy is  $k/2$ , it has its minimum kinetic energy at the origin.  
 (D) for small displacements from  $x = 0$ , the motion is simple harmonic.
9. A ball is hung vertically by a thread of length  $\ell$  from a point 'P' of an inclined wall that makes an angle ' $\alpha$ ' with the vertical. The thread with the ball is then deviated through a small angle ' $\beta$ ' ( $\beta > \alpha$ ) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are



- (A)  $2\sqrt{\frac{\ell}{g}} \left[ \sin^{-1} \left( \frac{\alpha}{\beta} \right) \right]$  (B)  $2\sqrt{\frac{\ell}{g}} \left[ \frac{\pi}{2} + \sin^{-1} \left( \frac{\alpha}{\beta} \right) \right]$   
 (C)  $2\sqrt{\frac{\ell}{g}} \left[ \cos^{-1} \left( \frac{\alpha}{\beta} \right) \right]$  (D)  $2\sqrt{\frac{\ell}{g}} \left[ \cos^{-1} \left( -\frac{\alpha}{\beta} \right) \right]$
10. If a SHM is given by  $y = (\sin \omega t + \cos \omega t) \text{ m}$ , which of the following statements are true?
- (A) The amplitude is 1m (B) The amplitude is  $\sqrt{2} \text{ m}$   
 (C) Time is considered from  $y = 1 \text{ m}$  (D) Time is considered from  $y = 0 \text{ m}$



11. The position of a particle at time  $t$  moving in  $x$ - $y$  plane is given by  $\vec{r} = (\hat{i} + 2\hat{j}) A \cos \omega t$ . Then, the motion of the particle is :  
 (A) on a straight line (B) on an ellipse (C) periodic (D) SHM
12. Three simple harmonic motions in the same direction having the same amplitude  $a$  and same period are superposed. If each differs in phase from the next by  $45^\circ$ , then, [IIT- 1999, 3/100]  
 (A) the resultant amplitude is  $(1+\sqrt{2})a$   
 (B) the phase of the resultant motion relative to the first is  $90^\circ$ .  
 (C) the energy associated with the resulting motion is  $(3+2\sqrt{2})$  times the energy associated with any single motion.  
 (D) the resulting motion is not simple harmonic.

## PART - IV : COMPREHENSION

### Comprehension-1

A 2kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of  $5 \text{ m/s}^2$  when the acceleration suddenly ceases at time  $t = 0$  and the car moves upward with constant speed. ( $g = 10 \text{ m/s}^2$ )

1. What is the angular frequency of oscillation of the block after the acceleration ceases?  
 (A)  $10\sqrt{2} \text{ rad/s}$  (B)  $20 \text{ rad/s}$  (C)  $20\sqrt{2} \text{ rad/s}$  (D)  $32 \text{ rad/s}$
2. The amplitude of the oscillations is  
 (A) 7.5 cm (B) 5 cm (C) 2.5 cm (D) 1 cm
3. The initial phase angle observed by an observer in the elevator, taking upward direction to be positive and positive extreme position to have  $\pi/2$  phase, is equal to  
 (A)  $-\pi/4 \text{ rad}$  (B)  $\pi/2 \text{ rad}$  (C)  $\pi \text{ rad}$  (D)  $3\pi/2 \text{ rad}$

### Comprehension-2

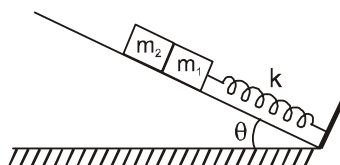
A particle of mass ' $m$ ' moves on a horizontal smooth line AB of length ' $a$ ' such that when particle is at any general point P on the line two forces act on it. A force  $\frac{mg(AP)}{a}$  towards A and another force  $\frac{2mg(BP)}{a}$  towards B.

4. Find its time period when released from rest from mid-point of line AB.  
 (A)  $T = 2\pi\sqrt{\frac{3a}{g}}$  (B)  $T = 2\pi\sqrt{\frac{a}{2g}}$  (C)  $T = 2\pi\sqrt{\frac{a}{g}}$  (D)  $T = 2\pi\sqrt{\frac{a}{3g}}$
5. Find the minimum distance of the particle from B during the motion.  
 (A)  $\frac{a}{6}$  (B)  $\frac{a}{4}$  (C)  $\frac{a}{3}$  (D)  $\frac{a}{8}$
6. If the force acting towards A stops acting when the particle is nearest to B then find the velocity with which it crosses point B.  
 (A)  $\frac{\sqrt{2ga}}{3}$  (B)  $\frac{\sqrt{2ga}}{6}$  (C)  $\frac{\sqrt{2ga}}{5}$  (D)  $\frac{\sqrt{ga}}{3}$



## Comprehension-3

Spring of spring constant  $k$  is attached with a block of mass  $m_1$  as shown in figure. Another block of mass  $m_2$  is placed against  $m_1$  and both masses lie on smooth incline plane.



7. Find the compression in the spring when the system is in equilibrium.
- (A)  $\frac{(m_1 + m_2)g \sin \theta}{2k}$  (B)  $\frac{(m_1 + m_2)g \sin \theta}{k}$
- (C)  $\frac{(m_1 + m_2)g}{k}$  (D)  $\frac{2(m_1 + m_2)g \sin \theta}{k}$
8. From the equilibrium position the blocks are pushed a further distance  $\frac{2}{k}(m_1 + m_2)g \sin \theta$  against the spring and released. Find the common speed of blocks when they separate.
- (A)  $\left( \sqrt{\frac{1}{3k}}(m_1 + m_2) \right) g \sin \theta$  (B)  $\left( \sqrt{\frac{2}{k}}(m_1 + m_2) \right) g \sin \theta$
- (C)  $\left( \sqrt{\frac{3}{k}}(m_1 + m_2) \right) g \sin \theta$  (D)  $\left( \sqrt{\frac{1}{k}}(m_1 + m_2) \right) g \sin \theta$

## Exercise-3

Marked Questions can be used as Revision Questions.

\* Marked Questions may have more than one correct option.

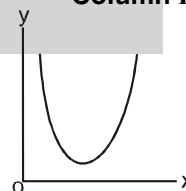
### PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

- 1\*. Function  $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$  represents SHM [JEE 2006, 5/184, -1]
- (A) for any value of  $A$ ,  $B$  and  $C$  (except  $C = 0$ ) (B) If  $A = -B$ ,  $C = 2B$ , amplitude =  $|B\sqrt{2}|$
- (C) If  $A = B$ ;  $C = 0$  (D) If  $A = B$ ;  $C = 2B$ , amplitude =  $|B|$
2. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. [IIT-JEE 2008, 6/163]

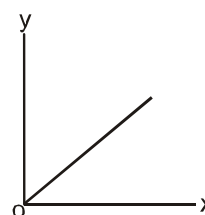
Column I

Column II

- (A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis)



- (B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.



**Resonance**  
Educating for better tomorrow

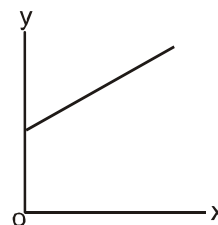
Corp. / Reg. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005  
Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)  
Toll Free : 1800 258 5555 | CIN : U80302RJ2007PLC024029

ADVSH - 41

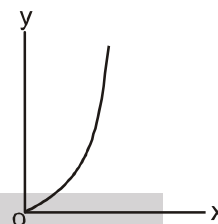




- (C) Range of a projectile (y-axis) as a function of its velocity (r) (x-axis) when projected at a fixed angle.

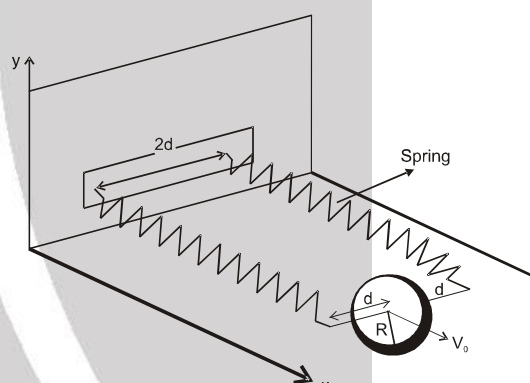


- (D) The square of the time period (y-axis) of a simple (s) pendulum as a function of its length (x-axis).



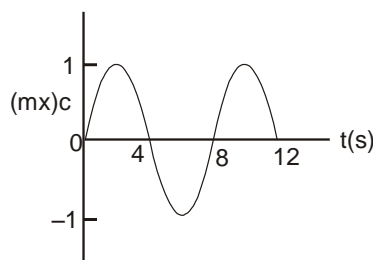
### Comprehension-1

A uniform thin cylindrical disk of mass  $M$  and radius  $R$  is attached to two identical massless springs of spring constant  $k$  which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance  $d$  from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is  $L$ . The disk is initially at its equilibrium position with its centre of mass (CM) at a distance  $L$  from the wall. The disk rolls without slipping with velocity  $\vec{V}_0 = V_0 \hat{i}$ . The coefficient of friction is  $\mu$ . Figure :



[JEE-2008, 3×4/163]

3. The net external force acting on the disk when its centre of mass is at displacement  $x$  with respect to its equilibrium position is
- (A)  $-kx$  (B)  $-2kx$  (C)  $-\frac{2kx}{3}$  (D)  $-\frac{4kx}{3}$
4. The centre of mass of the disk undergoes simple harmonic motion with angular frequency  $\omega$  equal to
- (A)  $\sqrt{\frac{k}{M}}$  (B)  $\sqrt{\frac{2k}{M}}$  (C)  $\sqrt{\frac{2k}{3M}}$  (D)  $\sqrt{\frac{4k}{3M}}$
5. The maximum value of  $V_0$  for which the disk will roll without slipping is
- (A)  $\mu g \sqrt{\frac{M}{k}}$  (B)  $\mu g \sqrt{\frac{M}{2k}}$  (C)  $\mu g \sqrt{\frac{3M}{k}}$  (D)  $\mu g \sqrt{\frac{5M}{2k}}$
6. The  $x$ - $t$  graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at  $t = 4/3$  s is

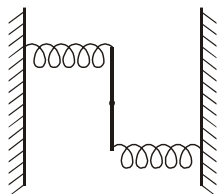


- (A)  $\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$  (B)  $-\frac{\pi^2}{32} \text{ cm/s}^2$  (C)  $\frac{\pi^2}{32} \text{ cm/s}^2$  (D)  $\frac{\sqrt{3}}{32} \pi^2 - \text{cm/s}^2$

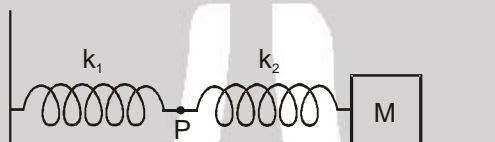




7. A uniform rod of length  $L$  and mass  $M$  is pivoted at the centre. Its two ends are attached to two springs of equal spring constants  $k$ . The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle  $\theta$  in one direction and released. The frequency of oscillation is : **[IIT-JEE 2009, 3/160, -1]**



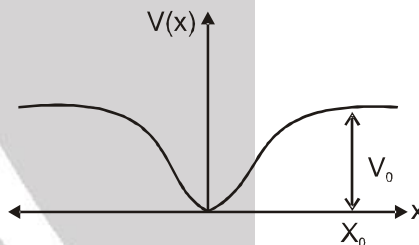
- (A)  $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$  (B)  $\frac{1}{2\pi} \sqrt{\frac{k}{M}}$  (C)  $\frac{1}{2\pi} \sqrt{\frac{6k}{M}}$  (D)  $\frac{1}{2\pi} \sqrt{\frac{24k}{M}}$
8. The mass  $M$  shown in the figure oscillates in simple harmonic motion with amplitude  $A$ . The amplitude of the point  $P$  is **[JEE 2009, 3/160, -1]**



- (A)  $\frac{k_1 A}{k_2}$  (B)  $\frac{k_2 A}{k_1}$  (C)  $\frac{k_1 A}{k_1 + k_2}$  (D)  $\frac{k_2 A}{k_1 + k_2}$

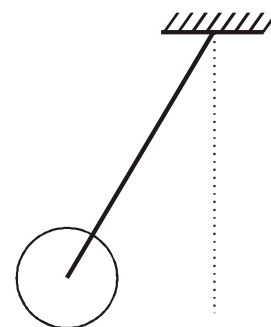
### Comprehension-2

When a particle of mass  $m$  moves on the  $x$ -axis in a potential of the form  $V(x) = kx^2$ , it performs simple harmonic motion. The corresponding time period is proportional to  $\sqrt{\frac{m}{k}}$ , as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of  $x = 0$  in a way different from  $kx^2$  and its total energy is such that the particle does not escape to infinity.



Consider a particle of mass  $m$  moving on the  $x$ -axis. Its potential energy is  $V(x) = \alpha x^4$  ( $\alpha > 0$ ) for  $|x|$  near the origin and becomes a constant equal to  $V_0$  for  $|x| \geq X_0$  (see figure) **[JEE 2010, 3×3/160, -1]**

9. If the total energy of the particle is  $E$ , it will perform periodic motion only if :  
 (A)  $E < 0$  (B)  $E > 0$  (C)  $V_0 > E > 0$  (D)  $E > V_0$
10. For periodic motion of small amplitude  $A$ , the time period  $T$  of this particle is proportional to :  
 (A)  $A \sqrt{\frac{m}{\alpha}}$  (B)  $\frac{1}{A} \sqrt{\frac{m}{\alpha}}$  (C)  $A \sqrt{\frac{\alpha}{m}}$  (D)  $\frac{1}{A} \sqrt{\frac{\alpha}{m}}$
11. The acceleration of this particle for  $|x| > X_0$  is :  
 (A) proportional to  $V_0$  (B) proportional to  $\frac{V_0}{mX_0}$   
 (C) proportional to  $\sqrt{\frac{V_0}{mX_0}}$  (D) zero

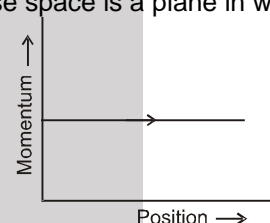


- 12\*. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' ( $R < L$ ) is attached at its center to the free end of the rod. Consider two ways the disk is attached: (case A) The disk is not free to rotate about its center and (case B) the disk is free to rotate about its center. The rod-disk system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? [JEE 2011, 4/160]

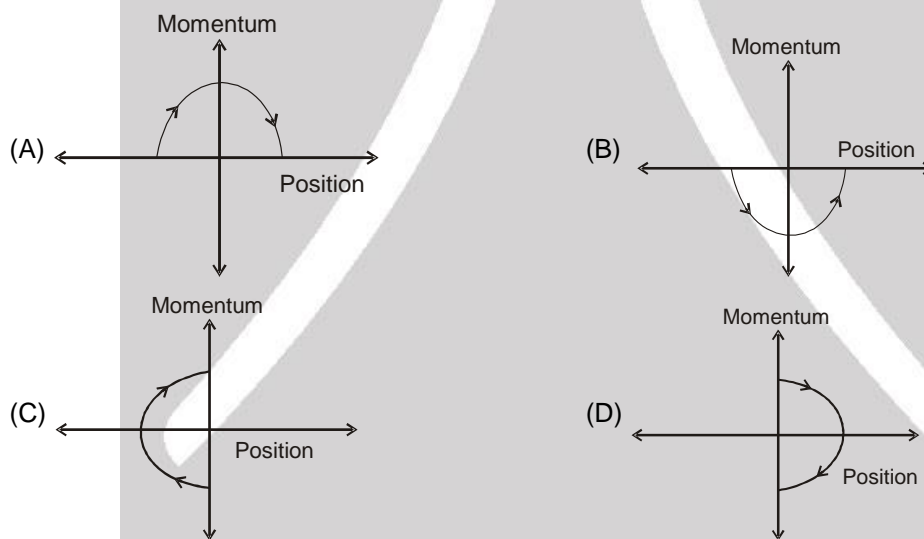
- (A) Restoring torque in case A = Restoring torque in case B  
 (B) Restoring torque in case A < Restoring torque in case B  
 (C) Angular frequency for case A > Angular frequency for case B.  
 (D) Angular frequency for case A < Angular frequency for case B.

### Comprehension-3

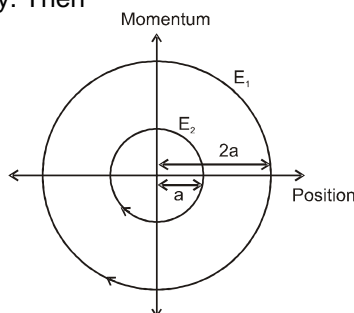
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is  $x(t)$  vs.  $p(t)$  curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum. Upwards (or to right) is positive and downwards (or to left) is negative. [JEE 2011, 3×3/160, -1]



13. The phase space diagram for a ball thrown vertically up from ground is :



14. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and  $E_1$  and  $E_2$  are the total mechanical energies respectively. Then

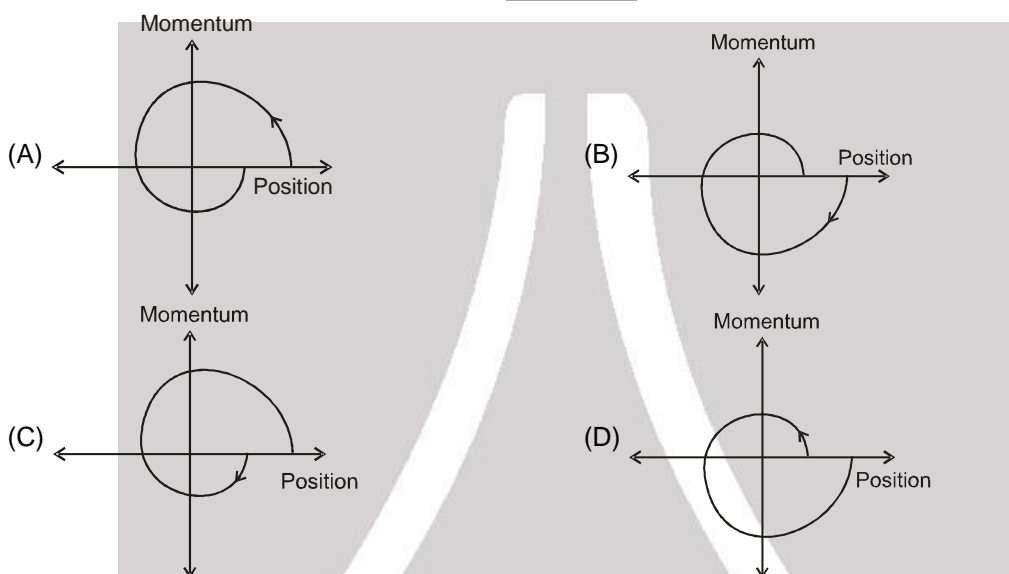
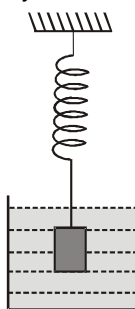


- (A)  $E_1 = \sqrt{2}E_2$  (B)  $E_1 = 2E_2$  (C)  $E_1 = 4E_2$  (D)  $E_1 = 16E_2$





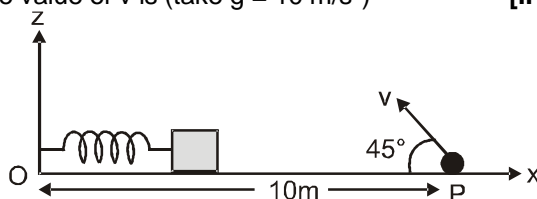
15. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is :



16. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction,  $x_1(t) = A \sin \omega t$  and  $x_2(t) = A \sin \left( \omega t + \frac{2\pi}{3} \right)$ . Adding a third sinusoidal displacement  $x_3(t) = B \sin (\omega t + \phi)$  brings the mass to a complete rest. The values of B and  $\phi$  are [JEE 2011, 3/160, -1]

- (A)  $\sqrt{2}A, \frac{3\pi}{4}$  (B)  $A, \frac{4\pi}{3}$  (C)  $\sqrt{3}A, \frac{5\pi}{6}$  (D)  $A, \frac{\pi}{3}$

17. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at  $t = 0$ . It then executes simple harmonic motion with angular frequency  $\omega = \frac{\pi}{3} \text{ rad/s}$ . Simultaneously at  $t = 0$ , a small pebble is projected with speed  $v$  from point P at an angle of  $45^\circ$  as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at  $t = 1\text{s}$ , the value of  $v$  is (take  $g = 10 \text{ m/s}^2$ ) [IIT-JEE-2012, Paper-1; 3/70, -1]



- (A)  $\sqrt{50} \text{ m/s}$  (B)  $\sqrt{51} \text{ m/s}$  (C)  $\sqrt{52} \text{ m/s}$  (D)  $\sqrt{53} \text{ m/s}$



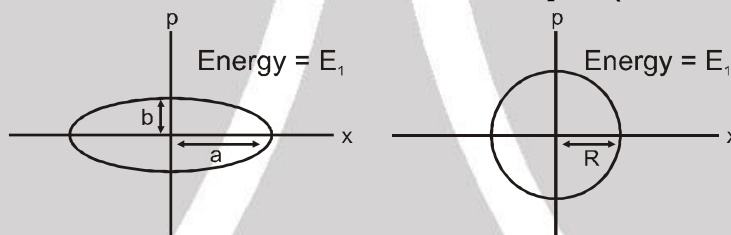
- 18\*. A particle of mass  $m$  is attached to one end of a mass-less spring of force constant  $k$ , lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time  $t = 0$  with an initial velocity  $u_0$ . When the speed of the particle is  $0.5 u_0$ , it collides elastically with a rigid wall. After this collision :

[JEE (Advanced) 2013, 4/60]

- (A) the speed of the particle when it returns to its equilibrium position is  $u_0$  .
- (B) the time at which the particle passes through the equilibrium position for the first time is  $t = \pi \sqrt{\frac{m}{k}}$  .
- (C) the time at which the maximum compression of the spring occurs is  $t = \frac{4\pi}{3} \sqrt{\frac{m}{k}}$  .
- (D) the time at which the particle passes throughout the equilibrium position for the second time is  $t = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$  .

19. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies  $\omega_1$  and  $\omega_2$  and have total energies  $E_1$  and  $E_2$ , respectively. The variations of their momenta  $p$  with positions  $x$  are shown in figures. If  $\frac{a}{b} = n^2$  and  $\frac{a}{R} = n$ , then the correct equation(s) is (are) :

[JEE (Advanced) 2015 ; P-1, 4/88, -2]



- (A)  $E_1 \omega_1 = E_2 \omega_2$       (B)  $\frac{\omega_2}{\omega_1} = n^2$       (C)  $\omega_1 \omega_2 = n^2$       (D)  $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

20. A particle of unit mass is moving along the  $x$ -axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and  $U_0$  are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

[JEE (Advanced) 2015 ; 8/88, -1]

Column-I

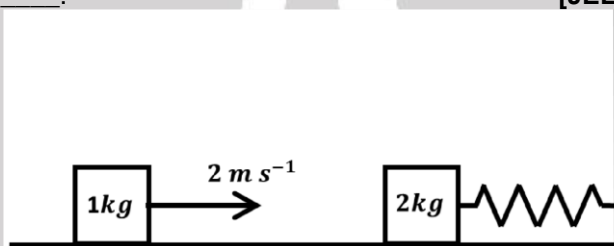
- (A)  $U_1(x) = \frac{U_0}{2} \left[ 1 - \left( \frac{x}{a} \right)^2 \right]^2$
- (B)  $U_2(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2$
- (C)  $U_3(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2 \exp \left[ - \left( \frac{x}{a} \right)^2 \right]$
- (D)  $U_4(x) = \frac{U_0}{2} \left[ \frac{x}{a} - \frac{1}{3} \left( \frac{x}{a} \right)^3 \right]$

Column-II

- (P) the force acting on the particle is zero at  $x = a$ .
- (Q) the force acting on the particle is zero at  $x = 0$ .
- (R) the force acting on the particle is zero at  $x = -a$ .
- (S) The particle experiences an attractive force towards  $x = 0$  in the region  $|x| < a$
- (T) The particle with total energy  $\frac{U_0}{4}$  can oscillate about the point  $x = -a$ .

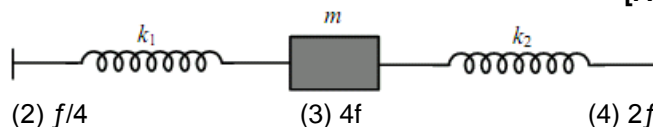


- 21\*. A block with mass  $M$  is connected by a massless spring with stiffness constant  $k$  to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude  $A$  about an equilibrium position  $x_0$ . Consider two cases : (i) when the block is at  $x_0$  ; and (ii) when the block is at  $x = x_0 + A$ . In both the cases, a particle with mass  $m$  ( $< M$ ) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass  $m$  is placed on the mass  $M$  ? [JEE (Advanced) 2016 ; P-2, 4/62, -2]
- (A) The amplitude of oscillation in the first case changes by a factor of  $\sqrt{\frac{M}{m+M}}$ , whereas in the second case it remains unchanged  
 (B) The final time period of oscillation in both the cases is same  
 (C) The total energy decreases in both the cases  
 (D) The instantaneous speed at  $x_0$  of the combined masses decreases in both the cases
22. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is  $2.0 \text{ Nm}^{-1}$  and the mass of the block is  $2.0 \text{ kg}$ . Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass  $1.0 \text{ kg}$  moving with a speed of  $2.0 \text{ ms}^{-1}$  collides elastically with the first block. The collision is such that the  $2.0 \text{ kg}$  block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is \_\_\_\_\_. [JEE (Advanced) 2018, P-1, 3/60]



## PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The maximum velocity of a particle, executing simple harmonic motion with an amplitude  $7 \text{ mm}$ , is  $4.4 \text{ m/s}$ . The period of oscillation is : [AIEEE 2006 ; 3/165, -1]  
 (1)  $100 \text{ s}$  (2)  $0.01 \text{ s}$  (3)  $10 \text{ s}$  (4)  $0.1 \text{ s}$
- 2\*. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency  $\omega$ . The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : [AIEEE 2006 ; 3/165, -1]  
 (1) at the highest position of the platform (2) at the mean position of the platform  
 (3) for an amplitude of  $\frac{g}{\omega^2}$  (4) for an amplitude of  $\frac{g^2}{\omega^2}$
3. The displacement of an object attached to a spring and executing simple harmonic motion is given by  $x = 2 \times 10^{-2} \cos \pi t$  metres. The time at which the maximum speed first occurs is : [AIEEE 2007 ; 3/120, -1]  
 (1)  $0.5 \text{ s}$  (2)  $0.75 \text{ s}$  (3)  $0.125 \text{ s}$  (4)  $0.25 \text{ s}$
4. A point mass oscillates along the  $x$ -axis according to the law  $x = x_0 \cos(\omega t - \pi/4)$ . If the acceleration of the particle is written as,  $a = A \cos(\omega t + \delta)$ , then : [AIEEE 2007 ; 3/120, -1]  
 (1)  $A = x_0, \delta = -\pi/4$  (2)  $A = x_0 \omega^2, \delta = -\pi/4$  (3)  $A = x_0 \omega^2, \delta = -\pi/4$  (4)  $A = x_0 \omega^2, \delta = 3\pi/4$
5. Two springs, of force constants  $k_1$  and  $k_2$ , are connected to a mass  $m$  as shown. The frequency of oscillation of mass is  $f$ . If both  $k_1$  and  $k_2$  are made four times their original values, the frequency of oscillation becomes: [AIEEE 2007 ; 3/120, -1]



- (1)  $f/2$  (2)  $f/4$  (3)  $4f$  (4)  $2f$



6. A particle of mass  $m$  executes simple harmonic motion with amplitude  $a$  and frequency  $\nu$ . The average kinetic energy during its motion from the position of equilibrium to the end is : **[AIEEE 2007; 3/120, -1]**  
 (1)  $\pi^2 ma^2 \nu^2$  (2)  $\frac{1}{4} ma^2 \nu^2$  (3)  $4\pi^2 ma^2 \nu^2$  (4)  $2\pi^2 ma^2 \nu^2$
- 7.\* If  $x$ ,  $v$  and  $a$  denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period  $T$ , then, which of the following does not change with time? **[AIEEE 2009 ; 3/120, -1]**  
 (1)  $\frac{aT}{x}$  (2)  $aT + 2\pi v$  (3)  $\frac{aT}{v}$  (4)  $a^2 T^2 + 4\pi^2 v^2$
8. A mass  $M$ , attached to a horizontal spring, executes SHM with a amplitude  $A_1$ . When the mass  $M$  passes through its mean position then a smaller mass  $m$  is placed over it and both of them move together with amplitude  $A_2$ . The ratio of  $\left(\frac{A_1}{A_2}\right)$  is : **[AIEEE - 2011, 4/120, -1]**  
 (1)  $\frac{M}{M+m}$  (2)  $\frac{M+m}{M}$  (3)  $\left(\frac{M}{M+m}\right)^{1/2}$  (4)  $\left(\frac{M+m}{M}\right)^{1/2}$
9. If a spring of stiffness ' $k$ ' is cut into two parts 'A' and 'B' of length  $\ell_A : \ell_B = 2 : 3$ , then the stiffness of spring 'A' is given by : **[AIEEE 2011, 11 May; 4/120, -1]**  
 (1)  $\frac{3k}{5}$  (2)  $\frac{2k}{5}$  (3)  $k$  (4)  $\frac{5k}{2}$
10. Two particles are executing simple harmonic motion of the same amplitude  $A$  and frequency  $\omega$  along the  $x$ -axis. Their mean position is separated by distance  $X_0 (X_0 > A)$ . If the maximum separation between them is  $(X_0 + A)$ , the phase difference between their motion is : **[AIEEE - 2011, 4/120, -1]**  
 (1)  $\pi/2$  (2)  $\pi/3$  (3)  $\pi/4$  (4)  $\pi/6$
11. If a simple pendulum has significant amplitude (up to a factor of  $1/e$  of original) only in the period between  $t = 0$ s to  $t = \tau$ s, then  $\tau$  may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with ' $b$ ' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds : **[AIEEE 2012 ; 4/120, -1]**  
 (1)  $\frac{0.693}{b}$  (2)  $b$  (3)  $\frac{1}{b}$  (4)  $\frac{2}{b}$
12. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to  $\alpha$  times its original magnitude, where  $\alpha$  equals. **[JEE (Main) 2013, 4/120]**  
 (1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6
13. A particle moves with simple harmonic motion in a straight line. In first  $\tau$  s, after starting from rest it travels a distance  $a$ , and in next  $\tau$  s it travels  $2a$ , in same direction, then : **[JEE (Main) 2014 ; 4/120, -1]**  
 (1) amplitude of motion is  $3a$  (2) time period of oscillations is  $8\tau$   
 (3) amplitude of motion is  $4a$  (4) time period of oscillations is  $6\tau$
14. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement  $d$ . Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **[JEE (Main) 2015 ; 4/120, -1]**
- (1)

(2)

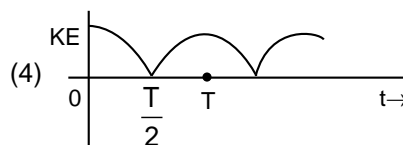
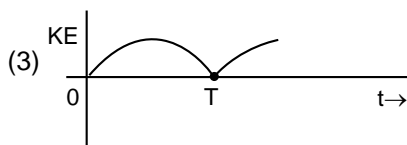
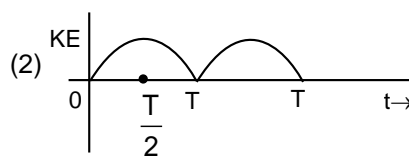
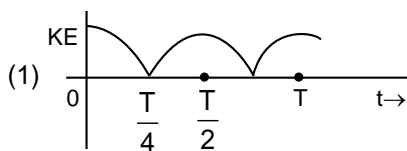
(3)

(4)
15. A particle performs simple harmonic motion with amplitude  $A$ . Its speed is trippled at the instant that it is at distance  $2A/3$  from equilibrium position. The new amplitude of the motion is. **[JEE (Main) 2016; 4/120, -1]**  
 (1)  $3A$  (2)  $\sqrt{3} A$  (3)  $\frac{7A}{3}$  (4)  $\frac{A}{3} \sqrt{41}$





16. A particle is executing simple harmonic motion with a time period  $T$ . At time  $= 0$ , it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like : [JEE (Main) 2017; 4/120, -1]



17. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of  $10^{12}$ /sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avagadro number =  $6.02 \times 10^{23}$  gm mole $^{-1}$ ) [JEE (Main) 2018; 4/120, -1]
- (1) 2.2 N/m (2) 5.5 N/m (3) 6.4 N/m (4) 7.1 N/m

## Answers

### EXERCISE-1

#### PART - I

#### Section (A) :

A-1. Amplitude = 5 m, Phase constant =  $\frac{\pi}{6}$ , Time period = 2 s, Maximum speed =  $5\pi$  m/s

A-2. (a) 2.0 cm,  $\frac{\pi}{50}$  s = 0.063 s, 100 N/m

(b) 1.0 cm,  $\sqrt{3}$  m/s, -100 m/s $^2$

A-3. (a) T/12, (b) T/8, (c) T/6, (d) T/4, (e) T/8

A-4.  $\frac{\sqrt{3}v_0}{2}$

A-5.  $x = (10\text{cm}) \sin \left[ \left( \frac{\pi}{3} \text{s}^{-1} \right) t + \frac{\pi}{6} \right]$ ,  $\frac{10}{9} \pi^2 \approx 11$  cm/s $^2$

A-6.  $\pm \frac{6}{5}$  cm =  $\pm 1.2$  cm from the mean position

#### Section (B) :

B-1.  $\pm 5$  cm

B-2. A = 0.06 m

#### Section (C) :

C-1. 0.1 N/m

C-2. 40 J

C-3.  $\frac{16}{10\pi^2} = 0.16$  Kg

C-4. (a)  $\frac{F}{k}$ ,  $2\pi\sqrt{\frac{M}{k}}$ , (b)  $\frac{F^2}{2k}$  (c)  $\frac{F^2}{2k}$

C-5. (a)  $2\pi\sqrt{\frac{m}{k_1+k_2}}$ ,  $k_{eq.} = k_1 + k_2$ ; (b)

$2\pi\sqrt{\frac{m}{k_1+k_2}}$ ,  $k_{eq.} = k_1 + k_2$ ; (c)

$2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$ ,  $k_{eq.} = \frac{k_1k_2}{k_1+k_2}$

Answers will remain same

C-6.  $\frac{M^2g^2}{2k}$ ,  $\frac{M^2g^2}{4k}$  and  $\frac{M^2g^2}{6k}$  from above,

time period =  $2\pi\sqrt{\frac{11M}{6k}}$

#### Section (D) :

D-1. 1 m

D-2. 0.25 m

D-3.  $\left( \frac{3600}{3601} \right)^2 g = 9.794 \text{m/s}^2$

D-4. (i)  $2T_0$

(ii) 3g upwards

#### Section (E) :

E-1. (a)  $T = 2\pi\sqrt{\frac{7\ell}{12g}}$

(b)  $2\pi\sqrt{\frac{2r}{g}}$

(c)  $2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$

(d)  $2\pi\sqrt{\frac{3r}{2g}}$

E-2. (a)  $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$ ,  $r/\sqrt{2}$

(b)  $2\pi\sqrt{\frac{L}{3g}}$ ,  $\frac{L}{2\sqrt{3}}$

#### Section (F) :

F-1. (a) 7 cm (b)  $\sqrt{37}$  cm = 6.1 cm (c) 5 cm

F-2.  $a\sqrt{4+2\sqrt{3}}$

F-3.  $2x^2 + \frac{y}{2} = 1$



**Section (G) :**

- G-1.** Both amplitude and energy of the particle can be maximum only in the case of resonance.  
For resonance to occur,  $\omega_1 = \omega_2$
- G-2.** (a) 0.3 s (b) 6.93 s (c) 3.4 s

**PART - II****Section (A) :**

- A-1.** (A)    **A-2.** (A)    **A-3.** (B)  
**A-4.** (A)    **A-5.** (A)    **A-6.** (C)  
**A-7.** (A)    **A-8.** (B)    **A-9.** (B)  
**A-10.** (B)

**Section (B) :**

- B-1.** (B)    **B-2.** (C)    **B-3.** (B)  
**B-4.** (C)    **B-5.** (A)    **B-6.** (C)

**Section (C) :**

- C-1.** (D)    **C-2.** (C)    **C-3.** (C)  
**C-4.** (A)    **C-5.** (D)    **C-6.** (D)  
**C-7.** (B)    **C-8.** (D)

**Section (D) :**

- D-1.** (A)    **D-2.** (D)    **D-3.** (C)  
**D-4.** (D)    **D-5.** (C)

**Section (E) :**

- E-1.** (D)    **E-2.** (B)

**Section (F) :**

- F-1.** (B)    **F-2.** (B)    **F-3.** (D)  
**F-4.** (B)

**Section (G) :**

- G-1.** (D)    **G-2.** (C)

**PART - III**

1. (A)  $\rightarrow$  r, (B)  $\rightarrow$  p, (C)  $\rightarrow$  q, (D)  $\rightarrow$  s  
 2. (A)  $\rightarrow$  p ; (B)  $\rightarrow$  q ; (C)  $\rightarrow$  p ; (D)  $\rightarrow$  s

**EXERCISE-2****PART - I**

1. (A)    2. (C)    3. (B)  
 4. (C)    5. (C)    6. (A)  
 7. (A)    8. (A)    9. (A)  
 10. (A)    11. (B)    12. (C)  
 13. (C)    14. (B)    15. (D)

**PART - II**

1. 10    2. 2    3. 2  
 4. 15    5. 2    6. 16  
 7. 3    8. 3    9. 12  
 10. 17    11. 15    12. 8  
 13. 15    14. 2    15. 8  
 16. 75

**PART - III**

1. (AD)    2. (ABC)    3. (BC)  
 4. (ABCDE)    5. (BD)    6. (ABCD)  
 7. (BCD)    8. (ABC)    9. (BD)  
 10. (BC)    11. (ACD)    12. (AC)

**PART - IV**

1. (A)    2. (C)    3. (D)  
 4. (D)    5. (A)    6. (B)  
 7. (B)    8. (C)

**EXERCISE-3****PART - I**

1. (ABD)  
 2. (A)  $\rightarrow$  (p); (B)  $\rightarrow$  (q, s) ; (C)  $\rightarrow$  (s) ; (D)  $\rightarrow$  (q)  
 3. (D)    4. (D)    5. (C)  
 6. (D)    7. (C)    8. (D)  
 9. (C)    10. (B)    11. (D)  
 12. (AD)    13. (D)    14. (C)  
 15. (B)    16. (B)    17. (A)  
 18. (AD)    19. (BD)  
 20. (A)  $\rightarrow$  P, Q, R, T ; (B)  $\rightarrow$  Q, S ; (C)  $\rightarrow$  P, Q, R, S ;  
 (D)  $\rightarrow$  P, R, T  
 21. (ABD)    22. 2.09

**PART - II**

1. (2)    2. (1, 3)    3. (1)  
 4. (4)    5. (4)    6. (1)  
 7. (1, 4)    8. (4)    9. (4)  
 10. (2)    11. (4)    12. (3)  
 13. (4)    14. (2)    15. (3)  
 16. (1)    17. (4)

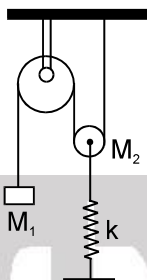




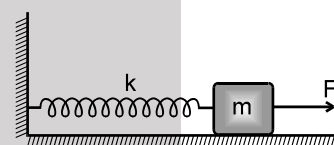
# High Level Problems (HLP)

## SUBJECTIVE QUESTIONS

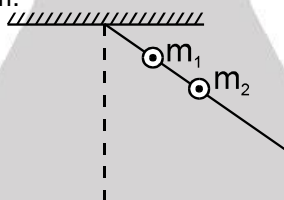
1. What would be the period of the free oscillations of the system shown here if mass  $M_1$  is pulled down a little force constant of the spring is  $k$ , mass of fixed pulley is negligible and movable pulley is smooth



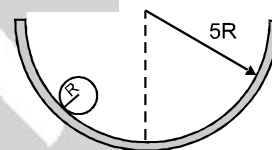
2. A constant force produces maximum velocity  $V$  on the block connected to the spring of force constant  $K$  as shown in the fig. When the force constant of spring becomes  $4K$ , then find maximum velocity of the block. Assume that initially the spring is in relaxed state.



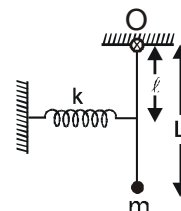
3. Two point masses  $m_1$  and  $m_2$  are fixed to a light rod hinged at one end. The masses are at distances  $\ell_1$  and  $\ell_2$  respectively from the hinge. Find the time period of oscillation (small amplitude) of this system in seconds if  $m_1 = m_2$ ,  $\ell_1 = 1\text{m}$ ,  $\ell_2 = 3\text{m}$ .



4. A solid sphere (radius =  $R$ ) rolls without slipping in a cylindrical vessel (radius =  $5R$ ). Find the angular frequency of small oscillations of the sphere in  $\text{s}^{-1}$ . Take  $R = \frac{1}{14}\text{m}$  and  $g = 10\text{ m/s}^2$ . (Axis of cylinder is fixed and horizontal).

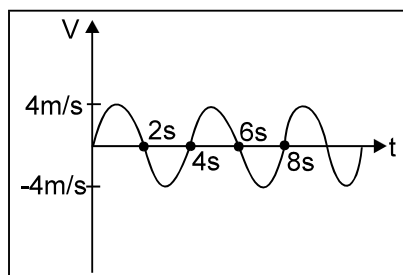


5. A particle of mass  $m$  is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point  $O$ . The spring is undeformed when the rod is vertical as shown in fig. If the period of oscillation of the system is  $\pi\sqrt{\frac{L}{n}}$ , when it is slightly displaced from its mean position then find  $n$ . Take  $k = \frac{9mgL}{\ell^2}$  and  $g = 10\text{m/s}^2$ .

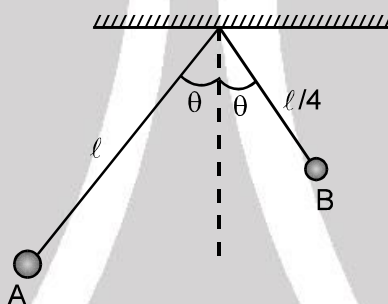




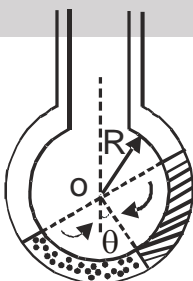
6. If velocity of a particle moving along a straight line changes sinusoidally with time as shown in the given graph. Find the average speed over time interval  $t = 0$  to  $t = 2(2n - 1)$  seconds,  $n$  being any positive integer.



7. Two simple pendulums A and B having lengths  $\ell$  and  $\ell/4$  respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle  $\theta$  is very small.



8. A particle of mass 'm' is moving in the x-y plane such that its x and y coordinate vary according to the law  $x = a \sin \omega t$  and  $y = a \cos \omega t$  where 'a' and ' $\omega$ ' are positive constants and 't' is time. Find
- equation of the path. Name the trajectory (path)
  - whether the particle moves in clockwise or anticlockwise direction
  - magnitude of the force on the particle at any time t.
9. Two non-viscous, incompressible and immiscible liquids of densities  $\rho$  and  $1.5\rho$  are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in fig. Each liquid occupies one-fourth the circumference of the tube.

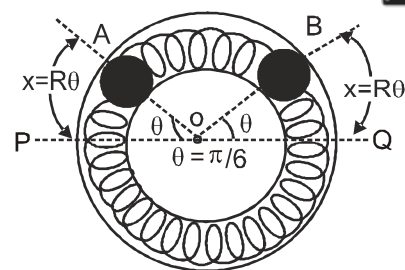


- Find the angle  $\theta$  that the radius to the interface makes with the vertical in equilibrium position.
- If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.





10. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of  $0.06\pi$  metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle  $\theta = \pi/6$  radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest.

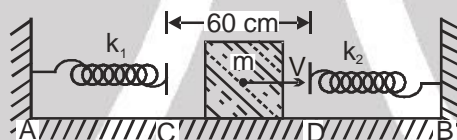


- Calculate the frequency of oscillation of ball B.
- Find the speed of ball A when A and B are at the two ends of the diameter PQ.
- What is the total energy of the system ?

[1993 ; 6M]

11. Two light springs of force constant  $k_1$  and  $k_2$  and a block of mass  $m$  are in one line AB on a smooth horizontal table such that one end of each spring is fixed to rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block. ( $k_1 = 1.8$  N/m,  $k_2 = 3.2$  N/m,  $m = 200$  g)

[1985 ; 6M]



12. Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass  $M$  is placed on it symmetrically. The friction co-efficient between each wheel and the plank is  $\mu$ . Find the frequency of oscillations, when plank is slightly displaced along its length and released.



13. **The Cubic Potential :** Consider a particle of mass  $m$  moving in one dimension under the influence of

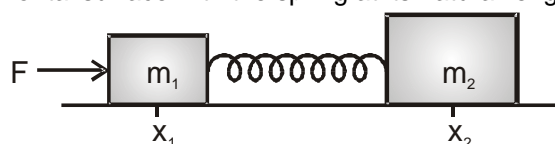
$$\text{potential energy } u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$$

Here  $\omega$ ,  $\delta$  and  $\alpha$  are real and positive.

- Sketch typical plots of  $u(x)$  and identify extrema if any.
- Consider the case where (in appropriate units) we have  $m = 1$ ,  $\omega = \sqrt{2}$ ,  $\alpha = 1$  and  $\delta = 1/2$ . Sketch the potential energy  $u(x)$ . If the total energy of the particle moving in this one-dimensional potential is  $E = 0$  (in same units), discuss the motion of the particle in terms of allowed regions, boundedness and periodicity.



14. Two blocks of masses  $m_1 = 1.0$  kg and  $m_2 = 2.0$  kg are connected by a massless elastic spring and are at rest on a smooth horizontal surface with the spring at its natural length.



A horizontal force of constant magnitude  $F = 6.0$  N is applied to the block  $m_1$  for a certain time  $t$  in which  $m_1$  suffers a displacement  $\Delta x_1 = 0.1$  m and  $\Delta x_2 = 0.05$  m. Kinetic energy of the system with respect to center of mass is 0.1 J. The force  $F$  is then withdrawn.

- Calculate  $t$ .
- Calculate the speed and the kinetic energy of the center of mass after the force is withdrawn.
- Calculate the energy stored in the system

## HLP Answers

- $T = 2\pi\sqrt{\frac{M_2 + 4M_1}{k}}$
- $V/2$
- $\pi$
- 5
- 25
- $\frac{8}{\pi}$  m/s
- $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$
- $x^2 + y^2 = a^2$ , circle
  - The particle moves in clock wise sense.
  - The magnitude of force =  $m\sqrt{a_x^2 + a_y^2} = m\omega^2 a$
- $\tan^{-1}\left(\frac{1}{5}\right)$
  - $2\pi\sqrt{\frac{R}{6.11}}$
- $f = \frac{1}{2\pi}\sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}$  Hz
  - $V = 0.0628$
  - $3.9 \times 10^{-4}$  J
- 2.82 s
- $2\pi\sqrt{\frac{\ell}{\mu g}}$
- between  $x = 0$  and  $x = \frac{3-\sqrt{3}}{2}$ .  $U$  is (-ve). So, K.E. is +ve.
- 0.26s
  - $0.52 \text{ ms}^{-1}$ , 0.40 J
  - 0.20 J





## Exercise-1

Marked Questions may have for Revision Questions.

चिह्नित प्रश्न दोहराने योग्य प्रश्न है।

### PART - I : SUBJECTIVE QUESTIONS

#### भाग - I : विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

#### SECTION (A) : EQUATION OF SHM सरल आवर्त गति का समीकरण

**A-1.** The equation of a particle executing SHM is  $x = (5 \text{ m}) \sin \left[ (\pi \text{ s}^{-1})t + \frac{\pi}{6} \right]$ . Write down the amplitude, initial phase constant, time period and maximum speed.

सरल आवर्त गति कर रहे एक कण का समीकरण  $x = (5 \text{ m}) \sin \left[ (\pi \text{ s}^{-1})t + \frac{\pi}{6} \right]$  है। आयाम, प्रारम्भिक कला नियतांक, आवर्तकाल व अधिकतम चाल लिखिए।

**Ans.** Amplitude = 5 m, Phase constant =  $\frac{\pi}{6}$ , Time period = 2 s, Maximum speed =  $5\pi$  m/s

आयाम = 5 m, कला नियतांक =  $\frac{\pi}{6}$ , आवर्त काल = 2 s, अधिकतम चाल =  $5\pi$  m/s

**Sol.**  $x = 5 \sin \left[ \pi t + \frac{\pi}{6} \right]$

Compare this equation with

$$x = A \sin (\omega t + \phi) \Rightarrow A = 5$$

$$\phi = \frac{\pi}{6} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2 \text{ sec}$$

$$V_{\max} = A\omega = 5\pi \text{ m/s}$$

**Sol.**  $x = 5 \sin \left[ \pi t + \frac{\pi}{6} \right]$

इस समीकरण की निम्न समीकरण के साथ तुलना करने पर

$$x = A \sin (\omega t + \phi) \Rightarrow A = 5$$

$$\phi = \frac{\pi}{6} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2 \text{ sec}$$

$$V_{\max} = A\omega = 5\pi \text{ m/s}$$

**A-2.** A particle having mass 10 g oscillates according to the equation  $x = (2.0 \text{ cm}) \sin [(100 \text{ s}^{-1})t + \pi/6]$ . Find (a) the amplitude, the time period and the force constant (b) the position, the velocity and the acceleration at  $t = 0$ .

10 g द्रव्यमान का एक कण समीकरण  $x = (2.0 \text{ cm}) \sin [(100 \text{ s}^{-1})t + \pi/6]$  के अनुसार दोलन करता है। ज्ञात करो (a) आयाम, आवर्तकाल व बल नियतांक (b)  $t = 0$  पर स्थिति, वेग व त्वरण।

**Ans.** (a) 2.0 cm,  $\frac{\pi}{50}$  s = 0.063 s, 100 N/m (b) 1.0 cm,  $\sqrt{3}$  m/s,  $-100 \text{ m/s}^2$



**Sol.**  $x = (2.0 \text{ cm}) \sin \left[ (100\text{s}^{-1})t + \frac{\pi}{6} \right]$

Compare this equation with

इसी समीकरण की तुलना करने पर

$$x = A \sin (\omega t + \phi) \Rightarrow A = 2.0 \text{ cm}$$

$$T = \frac{2\pi}{3} = \frac{2\pi}{100} = 0.0635$$

$$K = m\omega^2 = \frac{10}{100} 100^2 = 100 \text{ N/m}$$

$$x_{t=0} = 2.0 \sin \left[ 100 \times 0 + \frac{\pi}{6} \right] = 1 \text{ cm}$$

$$v = 200 \cos \left( 100t + \frac{\pi}{6} \right)$$

$$v_{t=0} = 200 \cos \left( 100 \times 0 + \frac{\pi}{6} \right)$$

$$= 100\sqrt{3} \text{ cm/s} = \sqrt{3} \text{ m/s}$$

$$a = -20000 \sin \left( 100 \times 0 + \frac{\pi}{6} \right) = 10000 \text{ cm/s}^2$$

$$= -100 \text{ m/s}^2$$

**A-3.** A simple harmonic motion has an amplitude  $A$  and time period  $T$ . Find the time required by it to travel directly from

एक सरल आवर्त गति का आयाम  $A$  व आवर्तकाल  $T$  है। इसके द्वारा निम्न दूरी तय करने के लिए आवश्यक समय ज्ञात कीजिए।

(a)  $x = 0$  to  $x = A/2$

(b)  $x = 0$  to  $x = \frac{A}{\sqrt{2}}$

(c)  $x = A$  to  $x = A/2$

(d)  $x = -\frac{A}{\sqrt{2}}$  to  $x = \frac{A}{\sqrt{2}}$

(e)  $x = \frac{A}{\sqrt{2}}$  to  $x = A$ .

(a)  $x = 0$  से  $x = A/2$  तक

(b)  $x = 0$  से  $x = \frac{A}{\sqrt{2}}$  तक

(c)  $x = A$  से  $x = A/2$  तक

(d)  $x = -\frac{A}{\sqrt{2}}$  से  $x = \frac{A}{\sqrt{2}}$  तक

(e)  $x = \frac{A}{\sqrt{2}}$  से  $x = A$  तक

**Ans.** (a)  $T/12$ , (b)  $T/8$ , (c)  $T/6$ , (d)  $T/4$ , (e)  $T/8$

**Sol.** (A)  $x = A \sin \left( \frac{2\pi}{T}t + \phi \right)$

$$0 = A \sin \left( \frac{2\pi}{T}0 + \phi \right)$$

$$\Rightarrow \frac{2\pi}{T}0 + \phi = 0 \Rightarrow \phi = 0$$

$$x = A \sin \left( \frac{2\pi t}{T} \right)$$

$$\frac{A}{2} = A \sin \left( \frac{2\pi t}{T} \right) \Rightarrow \frac{2\pi t}{T} = \frac{\pi}{6}$$

$$t = \frac{T}{12}$$







$$(D)x = A \sin \left[ \frac{2\pi t}{T} + \phi \right]$$

$$\frac{-A}{\sqrt{2}} = A \sin \left[ \frac{2\pi \cdot 0}{T} + \phi \right]$$

$$\phi = \frac{-\pi}{4}$$

$$x = A \sin \left[ \frac{2\pi t}{T} - \frac{\pi}{4} \right]$$

$$\frac{A}{\sqrt{2}} = A \sin \left[ \frac{2\pi t}{T} - \frac{\pi}{4} \right]$$

$$\frac{2\pi t}{T} - \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow t = \frac{T}{4}$$

- A-4.** A particle is executing SHM with amplitude  $A$  and has maximum velocity  $v_0$ . Find its speed when it is located at distance of  $\frac{A}{2}$  from mean position.

एक कण  $A$  आयाम  $v_0$  अधिकतम वेग से सरल आवर्त गति कर रहा है। माध्य अवस्था से  $\frac{A}{2}$  दूरी पर इसकी चाल ज्ञात करो।

**Ans.**  $\frac{\sqrt{3}v_0}{2}$

**Sol.**  $v_0 = A\omega \Rightarrow \omega = \frac{v_0}{A}$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v^2 = \left( \frac{v_0}{A} \right)^2 \left( A^2 - \frac{A^2}{2^2} \right)$$

$$v^2 = v_0^2 \frac{3}{4} \quad v = \frac{\sqrt{3}}{2} v_0$$

- A-5.** A particle executes simple harmonic motion with an amplitude of 10 cm and time period 6 s. At  $t = 0$  it is at position  $x = 5$  cm from mean position and going towards positive  $x$ -direction. Write the equation for the displacement  $x$  at time  $t$ . Find the magnitude of the acceleration of the particle at  $t = 4$  s.  
एक कण आयाम 10 cm व आवर्तकाल 6 s से सरल आवर्त गति कर रहा है।  $t = 0$  पर माध्य स्थिति से इसकी स्थिति  $x = 5$  cm है तथा यह धनात्मक  $x$ -दिशा में जा रहा है। समय  $t$  पर विस्थापन  $x$  के लिए समीकरण लिखो।  $t = 4$  s पर कण के त्वरण का परिमाण ज्ञात करो।

**Ans.**  $x = (10 \text{ cm}) \sin \left[ \left( \frac{\pi}{3} \text{ s}^{-1} \right) t + \frac{\pi}{6} \right], \frac{10}{9} \pi^2 \approx 11 \text{ cm/s}^2$

**Sol.**  $A = 10 \text{ cm}, \quad T = 6 \text{ sec}$

So अतः  $x = A \sin \left( \frac{2\pi}{T} t + \phi \right)$

$$= (10 \text{ cm}) \sin \left( \frac{2\pi}{6} t + \phi \right) = 10 \text{ cm} \sin \left( \frac{\pi}{3} t + \phi \right)$$

at  $t = 0$ ,  $x = 5 \text{ cm}$  पर

$$\Rightarrow 5 \text{ cm} = 10 \text{ cm} \sin (0 + \phi)$$

$$\Rightarrow \frac{1}{2} = \sin (\phi), \quad \phi = \frac{\pi}{6}$$



So, अतः  $x = (10 \text{ cm}) \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$

acceleration त्वरण  $a = -\omega^2 (10 \text{ cm}) \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = -\left(\frac{\pi}{3}\right)^2 \times (10 \text{ cm}) \sin\left(\frac{\pi}{3} \times 4 + \frac{\pi}{6}\right)$   
 $= \frac{\pi^2 \times 10}{9} \simeq 11 \text{ cm/s}^2$

**A-6.** A particle is executing SHM. Find the positions of the particle where its speed is 8 cm/s, If maximum magnitudes of its velocity and acceleration are 10 cm/s and 50 cm/s<sup>2</sup> respectively.

एक कण सरल आवर्त गति कर रहा है। कण की वह स्थितियाँ ज्ञात करें जहाँ कण की चाल 8 cm/s है, यदि कण की चाल तथा त्वरण के अधिकतम परिमाण क्रमशः 10 cm/s तथा 50 cm/s<sup>2</sup> है।

**Ans.**  $\pm \frac{6}{5} \text{ cm} = \pm 1.2 \text{ cm}$  from the mean position

माध्य स्थिति से  $\pm \frac{6}{5} \text{ cm} = \pm 1.2 \text{ cm}$

**Sol.**  $|v| = |A| \omega \Rightarrow 10 = |A| \omega$   
 $|a| = |A| \omega^2 \Rightarrow 50 = |A| \omega^2$   
 $\Rightarrow \omega = 5 \Rightarrow |A| = 2$   
 $v^2 = \omega^2 (A^2 - x^2)$   
 $8^2 = 5^2 (2^2 - x^2)$   
 $4 - x^2 = \frac{64}{25} \Rightarrow x^2 = \frac{36}{25} \Rightarrow x = \pm \frac{6}{5} \text{ cm}$

## SECTION (B) : ENERGY ऊर्जा

**B-1.** A particle performing SHM with amplitude 10cm. At What distance from mean position the kinetic energy of the particle is thrice of its potential energy?

एक कण 10cm आयाम के साथ सरल आवर्त गति कर रहा है। माध्य स्थिति से किस दूरी पर कण की गतिज ऊर्जा उसकी स्थिति ऊर्जा की तीन गुनी होगी ?

**Ans.**  $\pm 5 \text{ cm}$

**Sol.** According to question प्रश्नानुसार ,

$$\frac{1}{2} mv^2 = 3 \times \frac{1}{2} m\omega^2 x^2$$

$$\Rightarrow \frac{1}{2} m\omega^2(A^2 - x^2) = \frac{3}{2} m\omega^2 x^2$$

$$\Rightarrow A^2 = 4x^2 \Rightarrow x = \pm \frac{A}{2} = \pm \frac{10\text{cm}}{2} = \pm 5 \text{ cm}.$$

**B-2.** An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency of  $(25/\pi)$  Hz. At the position  $x = 0.04\text{m}$ , the object has kinetic energy of 0.5 J and potential energy 0.4 J. Find the amplitude of oscillations

एक 0.2 kg द्रव्यमान की वस्तु x-अक्ष के अनुदिश  $(25/\pi)$  Hz की आवृत्ति से सरल आवर्त गति करती है। स्थिति  $x = 0.04\text{m}$  पर वस्तु की गतिज ऊर्जा 0.5 J व स्थितिज ऊर्जा 0.4 J है। दोलन का आयाम ज्ञात करो।

**Ans.**  $A = 0.06 \text{ m}$

**Sol.** Total energy कुल ऊर्जा  $E = \frac{1}{2} m \omega^2 A^2$

$$0.5 + 0.4 = \frac{1}{2} \times 0.2 \left(2\pi \times \frac{25}{\pi}\right)^2 A^2$$

$$\Rightarrow 9 = (50)^2 A^2 \Rightarrow A = 0.06 \text{ m}$$



## SECTION (C) : SPRING MASS SYSTEM

## स्प्रिंग द्रव्यमान निकाय

**C-1.** A spring mass system has a time period of 2 second. What should be the spring constant of spring if the mass of the block is 10 grams?

एक स्प्रिंग द्रव्यमान निकाय का आवर्तकाल 2 सेकण्ड है। यदि ब्लॉक का द्रव्यमान 10 ग्राम हो, तो स्प्रिंग का स्प्रिंग नियतांक क्या होना चाहिए।

**Ans.** 0.1 N/m

**Sol.**  $T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow 2 = 2\pi\sqrt{\frac{10/1000}{K}} \Rightarrow K = 0.1 \text{ N/m.}$

**C-2.** A body of mass 2 kg suspended through a vertical spring executes simple harmonic motion of period 4s. If the oscillations are stopped and the body hangs in equilibrium, find the potential energy stored in the spring.

एक ऊर्ध्वाधर स्प्रिंग से लटकी 2 kg द्रव्यमान की वस्तु 4s के आवर्त काल से सरल आवर्त गति करती है। यदि दोलन रोक दिये जायें व वस्तु साम्यावस्था में लटकी रहती है, तो स्प्रिंग में संग्रहित स्थितिज ऊर्जा ज्ञात करो।

**Ans.** 40 J

**Sol.**  $K = m\omega^2 \Rightarrow K = m \left( \frac{2\pi}{T} \right)^2$

$$K = \frac{4\pi^2 m}{T^2}$$

$$mg = Kx \text{ equilibrium साम्यवस्था}$$

$$x = \frac{mg}{K} \Rightarrow U = \frac{1}{2} Kx^2$$

$$= \frac{1}{2} K \frac{m^2 g^2}{K^2} = \frac{m^2 g^2 T^2}{2 \times 4\pi^2 m} = \frac{2^2 \times 10^2 \times 4^2}{2 \times 4 \times 10 \times 2} = 40 \text{ J}$$

**C-3.** A vertical spring-mass system with lower end of spring is fixed, made to undergo small oscillations. If the spring is stretched by 25cm, energy stored in the spring is 5J. Find the mass of the block if it makes 5 oscillations each second.

एक ऊर्ध्वाधर स्प्रिंग द्रव्यमान निकाय जिसमें स्प्रिंग का नीचला सिरा स्थिर है, में अल्प दोलन करवाये जाते हैं। यदि स्प्रिंग को 25 cm से खींचा जाता है तो स्प्रिंग 5 J ऊर्जा संग्रहित करती है। यदि गुटका प्रत्येक सेकण्ड में 5 दोलन करता है तो गुटके का द्रव्यमान ज्ञात करो।

**Ans.**  $\frac{16}{10\pi^2} = 0.16 \text{ Kg}$

**Sol.**  $\frac{1}{2} Kx^2 = 5$

$$\frac{1}{2} k \times \left( \frac{25}{100} \right)^2 = 5$$

$$k = 160$$

$$\omega = 10\pi = \sqrt{\frac{K}{m}}$$

$$m = \frac{K}{100\pi^2}$$

$$m = \frac{160}{100\pi^2} = \frac{16}{10\pi^2} = 0.16 \text{ kg}$$

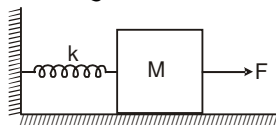


**C-4.#** A spring mass system is shown in figure, spring is initially unstretched. A man starts pulling the block with constant force  $F$ . Find

- The amplitude and the time period of motion of the block
- The K.E. of the block at mean position
- The energy stored in the spring when the block passes through the mean position

चित्र में एक स्प्रिंग द्रव्यमान निकाय दिखाया गया है, स्प्रिंग प्रारम्भ में अविस्तारित है। एक व्यक्ति नियत बल  $F$  से वस्तु को खींचना प्रारम्भ करता है तो ज्ञात करो

- वस्तु की गति का आयाम व आवर्तकाल
- माध्य अवस्था पर वस्तु की गतिज ऊर्जा
- स्प्रिंग में संग्रहित ऊर्जा जब ब्लॉक माध्य अवस्था से गुजरता है।



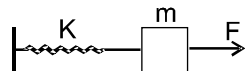
**Ans.** (a)  $\frac{F}{k}$ ,  $2\pi\sqrt{\frac{M}{k}}$ , (b)  $\frac{F^2}{2k}$  (c)  $\frac{F^2}{2k}$

**Sol.** (a) Initial position of block is an extreme position. ब्लॉक की प्रारम्भिक अवस्था एक सीमान्त स्थिति है

At equilibrium साम्यवस्था पर  $F = KA \Rightarrow A = F/K$

Time period आवर्तकाल

$$T = 2\pi\sqrt{\frac{m}{K}}$$



(b) Kinetic energy at mean position माध्य अवस्था पर गतिज ऊर्जा

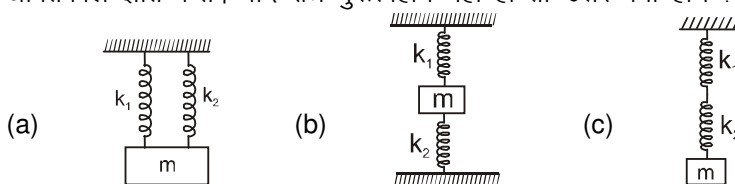
$$\text{K.E.} = \frac{1}{2} K A^2 = \frac{1}{2} K \left( \frac{F}{K} \right)^2 = \frac{F^2}{2K}$$

(c) Total energy of the block. ब्लॉक की कुल ऊर्जा

$$\frac{1}{2} K A^2 = \frac{1}{2} K \left( \frac{2F}{K} \right)^2 = \frac{F^2}{2K}$$

**C-5.#** Three spring mass systems are shown in figure. Assuming gravity free space, find the time period of oscillations in each case. What should be the answer if space is not gravity free?

चित्रानुसार तीन स्प्रिंग द्रव्यमान निकाय दिखाये गये हैं तथा गुरुत्वीय मुक्त क्षेत्र मानना है। प्रत्येक स्थिति में दोलन का आवर्तकाल ज्ञात करो। यदि क्षेत्र गुरुत्वहीन नहीं हो तो उत्तर क्या होंगे?



**Ans.** (a)  $2\pi\sqrt{\frac{m}{k_1+k_2}}$ ,  $k_{eq.} = k_1 + k_2$ ; (b)  $2\pi\sqrt{\frac{m}{k_1+k_2}}$ ,  $k_{eq.} = k_1 + k_2$ ; (c)  $2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$ ,  $k_{eq.} = \frac{k_1k_2}{k_1+k_2}$

Answers will remain same उत्तर समान ही होंगे

**Sol.** (a) When spring is stretched by  $x$  then restoring force.

जब स्प्रिंग को  $x$  से खींचा जाता है, तो प्रत्यानयन बल

$$F = K_1 x + K_2 x$$

$$F = K_{eq} x$$

$$K_{eq} x = K_1 x + K_2 x$$

$$K_{eq} = K_1 + K_2$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$$



(b) When block is displaced by  $x$  from mean position then restoring force.

जब ब्लॉक को माध्य अवस्था से  $x$  खींचा जाता है तब प्रत्यानयन बल

$$F = K_1 x + K_2 x$$

$$K_{eq} x = K_1 x + K_2 x$$

$$K_{eq} = K_1 + K_2$$

$$T = 2\pi \sqrt{\frac{m}{K_{eq}}} = 2\pi \sqrt{\frac{m}{K_1 + K_2}}$$

(c) When block is displaced by  $x$  and extension in upper spring is  $x_1$ , extension in lower spring is  $x_2$  जब ब्लॉक को  $x$  विस्थापित किया जाता है तथा ऊपरी स्प्रिंग में विस्तार  $x_1$ , तथा निचली स्प्रिंग में विस्तार  $x_2$  है।

$$\text{then तब } F = K_1 x_1 \Rightarrow x_1 = \frac{F}{K_1}$$

$$F = K_2 x_2 \Rightarrow x_2 = \frac{F}{K_2}$$

$$F = K_{eq} x \Rightarrow x = \frac{F}{K_{eq}}$$

$$x = x_1 + x_2 \Rightarrow \frac{F}{K_{eq}} = \frac{F}{K_1} + \frac{F}{K_2} \Rightarrow K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$$

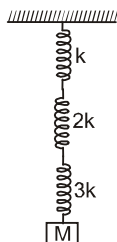
$$T = 2\pi \sqrt{\frac{m}{K_{eq}}} = 2\pi \sqrt{\frac{m(K_1 + K_2)}{K_1 K_2}}$$

When space is not gravity free then answers do not change as time period of spring mass system is independent of gravity.

जब वातावरण गुरुत्व मुक्त नहीं है तो उत्तर नहीं बदलता है क्योंकि स्प्रिंग द्रव्यमान निकाय का आवर्तकाल गुरुत्व से स्वतंत्र होता है।

**C-6.** Spring mass system is shown in figure. Find the elastic potential energy stored in each spring when block is at its mean position. Also find the time period of vertical oscillations. The system is in vertical plane.

चित्र में स्प्रिंग द्रव्यमान निकाय दिखाया गया है। जब ब्लॉक माध्यवस्था पर है, तब प्रत्येक स्प्रिंग में संग्रहित प्रत्यास्थ स्थितिज ऊर्जा ज्ञात करो। पिण्ड के ऊर्ध्वाधर दोलन का आवर्तकाल भी ज्ञात करो।



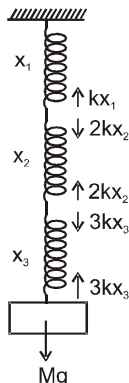
**Ans.**  $\frac{M^2 g^2}{2k}$ ,  $\frac{M^2 g^2}{4k}$  and  $\frac{M^2 g^2}{6k}$  from above, time period  $= 2\pi \sqrt{\frac{11M}{6k}}$

ऊपर से  $\frac{M^2 g^2}{2k}$ ,  $\frac{M^2 g^2}{4k}$  तथा  $\frac{M^2 g^2}{6k}$ ; आवर्तकाल  $= 2\pi \sqrt{\frac{11M}{6k}}$





**Sol.** Let at equilibrium, elongation in springs are  $x_1$ ,  $x_2$  and  $x_3$  then by force balance  
माना कि साम्यवस्था में स्प्रिंगों में विस्तार  $x_1$ ,  $x_2$  तथा  $x_3$  है तो बल संतुलन से



$$3kx_3 = Mg = 2kx_2 = kx_1$$

$$\Rightarrow x_1 = \frac{Mg}{k}; x_2 = \frac{Mg}{2k}; x_3 = \frac{Mg}{3k}$$

So potential energy stored in each spring

अतः प्रत्येक स्प्रिंग में संचित ऊर्जा

$$(P.E.)_1 = \frac{1}{2} kx_1^2 = \frac{M^2 g^2}{2k}$$

$$(P.E.)_2 = \frac{1}{2} 2kx_2^2 = \frac{M^2 g^2}{4k}$$

$$(P.E.)_3 = \frac{1}{2} 3kx_3^2 = \frac{M^2 g^2}{6k}$$

and Net spring constant और कुल बल नियतांक  $= \frac{1}{1/k + 1/2k + 1/3k} = \frac{6k}{11}$

So Time period इसलिये आवर्तकाल  $= 2\pi \sqrt{\frac{M}{k_{eq}}} = 2\pi \sqrt{\frac{11}{6} \frac{M}{k}}$

### Section (D) : Simple Pendulum सरल लोलक

**D-1.** Find the length of seconds pendulum at a place where  $g = \pi^2 \text{ m/s}^2$ .  
उस स्थान पर सेकण्ड लोलक की लम्बाई ज्ञात करो, जहाँ  $g = \pi^2 \text{ m/s}^2$ ।

**Ans** 1 m

**Sol.**  $T = 2\pi \sqrt{\frac{\ell}{g}} \quad 2 = 2\pi \sqrt{\frac{\ell}{g}} \quad \Rightarrow \ell = 1 \text{ m}$

**D-2.** Instantaneous angle (in radian) between string of a simple pendulum and vertical is given by

$$\theta = \frac{\pi}{180} \sin 2\pi t. \text{ Find the length of the pendulum if } g = \pi^2 \text{ m/s}^2$$

एक सरल लोलक की डोरी के द्वारा ऊर्ध्वाधर के साथ बनाया गया तात्क्षणिक कोण (रेडियन में) समय पर

$$\theta = \frac{\pi}{180} \sin 2\pi t \text{ के अनुसार निर्भर करता है। तो लोलक की लम्बाई ज्ञात करो यदि } g = \pi^2 \text{ m/s}^2 \text{ है।}$$

**Ans.** 0.25 m



**Sol.** Time period आवर्तकाल  $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1 = 2\pi\sqrt{\frac{\ell}{g}}$

$$= 2\pi\sqrt{\frac{\ell}{\pi^2}} \Rightarrow 1 = 2\sqrt{\ell} \Rightarrow \ell = 0.25 \text{ m.}$$

**D-3.** A pendulum clock is accurate at a place where  $g = 9.8 \text{ m/s}^2$ . Find the value of  $g$  at another place where clock becomes slow by 24 seconds in a day (24 hrs).

एक लोलक घड़ी उस स्थान पर सही समय देती है जहाँ  $g = 9.8 \text{ m/s}^2$  है। यह किसी दूसरे स्थान पर ले जाई जाती है, जहाँ यह 24 घण्टे में 24 सैकण्ड पीछे रह जाती है तो इस नये स्थान पर  $g$  का मान ज्ञात करो।

**Ans.**  $\left(\frac{3600}{3601}\right)^2 g = 9.794 \text{ m/s}^2$

**Sol.**  $T = 2\pi\sqrt{\frac{\ell}{g}} \quad T' = 2\pi\sqrt{\frac{\ell}{g'}}$

$$\frac{T}{T'} = \sqrt{\frac{g'}{g}} \Rightarrow \frac{24 \times 3600}{24 \times 3600 + 24} = \sqrt{\frac{g'}{g}} \Rightarrow g' = \left(\frac{3600}{3601}\right)^2 g = 9.794 \text{ m/s}^2$$

**D-4.** A pendulum is suspended in a lift and its period of oscillation is  $T_0$  when the lift is stationary.

एक लोलक एक लिफ्ट में लटका है व इसका दोलन काल  $T_0$  है, जब लिफ्ट स्थिर है।

(i) What will be the period  $T$  of oscillation of pendulum, if the lift begins to accelerate downwards with an acceleration equal to  $\frac{3g}{4}$ ?

लोलक का दोलन काल  $T$  क्या होगा, यदि लिफ्ट नीचे की ओर  $\frac{3g}{4}$  त्वरण से चलना प्रारम्भ करे ?

(ii) What must be the acceleration of the lift for the period of oscillation of the pendulum to be  $\frac{T_0}{2}$ ?

लोलक का दोलनकाल  $\frac{T_0}{2}$  होने के लिए लिफ्ट का त्वरण क्या होना चाहिए ?

**Ans.** (i)  $2T_0$  (ii)  $3g$  upwards ऊपर की ओर

**Sol.** (i)  $T_0 = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} \quad g_{\text{eff}} = g - \frac{3g}{4} = \frac{g}{4}$

So, time period अतः आवर्त काल  $T = 2\pi\sqrt{\frac{\ell}{g/4}} \Rightarrow T = 2T_0$

(ii)  $\frac{T_0}{2} = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}}, \quad \frac{2\pi}{2}\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}}$

$g_{\text{eff}} = 4g \Rightarrow a = 3g$  upwards ऊपर की ओर

## SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

### पिण्ड लोलक व मरोड़ी लोलक

**E-1.** Compound pendulums are made of :

- A rod of length  $\ell$  suspended through a point located at distance  $\ell/4$  from centre of rod.
- A ring of mass  $m$  and radius  $r$  suspended through a point on its periphery.
- A uniform square plate of edge  $a$  suspended through a corner.
- A uniform disc of mass  $m$  and radius  $r$  suspended through a point  $r/2$  away from the centre.

Find the time period in each case.





भौतिक लोलक निम्न से बनाया जाता है

- (a) एक  $\ell$  लम्बाई की छड़ जो छड़ के केन्द्र से  $\ell/4$  दूरी पर लटकी है।  
 (b) द्रव्यमान  $m$  व त्रिज्या  $r$  की एक वलय जो इसकी परिधि पर किसी बिन्दु से लटकी है।  
 (c) एक कोने से लटकी हुई  $a$  भुजा की एक समरूप वर्गाकार प्लेट  
 (d) केन्द्र से  $r/2$  दूरी पर लटकी हुई  $m$  द्रव्यमान व  $r$  त्रिज्या की एक समरूप चकती।  
 प्रत्येक स्थिति में अल्प दोलों का आवर्तकाल ज्ञात कीजिए।

**Ans.** (a)  $T = 2\pi\sqrt{\frac{7\ell}{12g}}$  (b)  $2\pi\sqrt{\frac{2r}{g}}$  (c)  $2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$  (d)  $2\pi\sqrt{\frac{3r}{2g}}$

**Sol.** (a) Time period of compound pendulum is भौतिक लोलक का आवर्तकाल  $T = 2\pi\sqrt{\frac{I_{cm} + md^2}{mg\ell}}$

For rod of length  $\ell$ ,  $\ell$  लम्बाई की छड़ के लिए

$$T = 2\pi\sqrt{\frac{\frac{m\ell^2}{12} + m\left(\frac{\ell}{4}\right)^2}{mg\ell/4}} = 2\pi\sqrt{\frac{m\ell^2(4+3)}{48mg\ell/4}}$$

$$T = 2\pi\sqrt{\frac{7\ell}{12g}} \quad \text{Ans.}$$

(b)  $T = 2\pi\sqrt{\frac{mr^2 + mr^2}{mgr}}$

$$T = 2\pi\sqrt{\frac{2r}{g}} \quad \text{Ans.}$$

(c)  $T = 2\pi\sqrt{\frac{\frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2}{mg\frac{a}{\sqrt{2}}}}$

$$T = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}} \quad \text{Ans.}$$

(d)  $T = 2\pi\sqrt{\frac{\frac{mr^2}{2} + m\left(\frac{r}{2}\right)^2}{mg\frac{r}{2}}}$

$$T = 2\pi\sqrt{\frac{3r}{2g}} \quad \text{Ans.}$$

**E-2.** Two compound pendulums are made of :

- (a) A disc of radius  $r$  and  
 (b) A uniform rod of length  $L$ . Find the minimum possible time period and distance between centre and point of suspension in each case.  
 दो भौतिक लोलक निम्न से बनाये गये हैं।  
 (a)  $r$  त्रिज्या की एक चकती  
 (b)  $L$  लम्बाई की एक समान छड़  
 अल्प दोलों के लिए न्यूनतम संभव आवर्तकाल ज्ञात कीजिए तथा न्यूनतम संभव आवर्तकाल के लिए छिद्र की केन्द्र से दूरी भी ज्ञात करो।

**Ans.** (a)  $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$ ,  $r/\sqrt{2}$  (b)  $2\pi\sqrt{\frac{L}{3g}}$ ,  $\frac{L}{2\sqrt{3}}$



- Sol.** (a) For minimum time period  $\ell = K$ , where  $\ell$  is distance between centre and point of suspension  
न्यूनतम आवर्तकाल के लिए  $\ell = K$ , जहाँ  $\ell$  केन्द्र तथा निलम्बन बिन्दु के बीच की दूरी है

$$\text{Moment of inertia about centre} = \frac{mr^2}{2} = mK^2 \Rightarrow K = \frac{r}{\sqrt{2}} = \ell$$

$$\text{केन्द्र के परितः जड़त्वाघूर्ण} = \frac{mr^2}{2} = mK^2 \Rightarrow K = \frac{r}{\sqrt{2}} = \ell$$

$$\text{minimum time period } T_{\min} = 2\pi \sqrt{\frac{2k}{g}} = 2\pi \sqrt{\frac{r\sqrt{2}}{g}}$$

$$\text{न्यूनतम आवर्तकाल } T_{\min} = 2\pi \sqrt{\frac{2k}{g}} = 2\pi \sqrt{\frac{r\sqrt{2}}{g}}$$

$$(b) \text{ Moment of inertia about centre} = \frac{mL^2}{12} = mK^2$$

$$(b) \text{ केन्द्र के परितः जड़त्वाघूर्ण} = \frac{mL^2}{12} = mK^2$$

$$\Rightarrow K = \frac{L}{2\sqrt{3}} = \ell \quad \text{where } \ell \text{ is distance between centre and point of suspension}$$

जहाँ  $\ell$  केन्द्र तथा निलम्बन बिन्दु के बीच की दूरी है

$$\text{Minimum time period } T = 2\pi \sqrt{\frac{2k}{g}} = 2\pi \sqrt{\frac{L}{\sqrt{3}g}}$$

$$\text{न्यूनतम आवर्तकाल } T = 2\pi \sqrt{\frac{2k}{g}} = 2\pi \sqrt{\frac{L}{\sqrt{3}g}}$$

## SECTION (F) : SUPERPOSITION OF SHM सरल आवर्त गति का अध्यारोपण

- F-1.** A particle is subjected to two SHM's simultaneously

$$X_1 = a_1 \sin \omega t \text{ and } X_2 = a_2 \sin(\omega t + \phi)$$

Where  $a_1 = 3.0 \text{ cm}$ ,  $a_2 = 4.0 \text{ cm}$ .

Find resultant amplitude if the phase difference  $\phi$  has values (a)  $0^\circ$  (b)  $60^\circ$  (c)  $90^\circ$

निम्न दो सरल आवर्त गतियों के अधीन एक कण गतिशील है

$$X_1 = a_1 \sin \omega t \text{ तथा } X_2 = a_2 \sin(\omega t + \phi)$$

जहाँ  $a_1 = 3.0 \text{ cm}$ ,  $a_2 = 4.0 \text{ cm}$

दोनों गतियों का परिणामी आयाम ज्ञात करो यदि इन गतियों के मध्य कलान्तर (a)  $0^\circ$ , (b)  $60^\circ$ , (c)  $90^\circ$  हो।

- Ans.** (a) 7 cm (b)  $\sqrt{37} \text{ cm} = 6.1 \text{ cm}$  (c) 5 cm

**Sol.**  $A_r = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \theta}$

$$(a) \quad A_r = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 0} = 7 \text{ cm}$$

$$(b) \quad A_r = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 60} = \sqrt{37} \text{ cm} = 6.1 \text{ cm}$$

$$(c) \quad A_r = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 90} = 5 \text{ cm}$$

- F-2.** A particle is subjected to three SHM's in same direction simultaneously each having equal amplitude  $a$  and equal time period. The phase of the second motion is  $30^\circ$  ahead of the first and the phase of the third motion is  $30^\circ$  ahead of the second. Find the amplitude of the resultant motion.

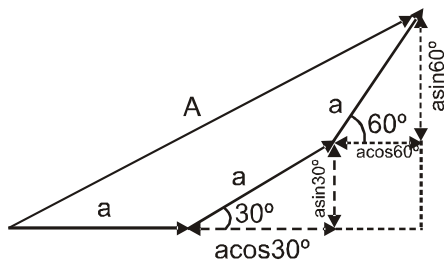
तीन सरल आवर्त गतियों जो समान आयाम  $a$  तथा समान आवर्त काल के साथ एक दिशा में हो रही हैं, के अधीन एक कण गति कर रहा है। द्वितीय गति, प्रथम से  $30^\circ$  कलान्तर आगे तथा तृतीय गति, द्वितीय गति से  $30^\circ$  कलान्तर आगे है, तो परिणामी गति का आयाम ज्ञात करो।

**Ans.**  $a\sqrt{4 + 2\sqrt{3}}$





Sol.



From phasor diagram, कला आरेख से

$$A = \sqrt{\left(a + \frac{\sqrt{3}a}{2} + \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2} + \frac{a}{2}\right)^2}$$

$$= \sqrt{\left(\frac{3}{2}a + \frac{\sqrt{3}a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2} + \frac{a}{2}\right)^2}$$

$$A = \frac{a}{2} \sqrt{(3 + \sqrt{3})^2 + (\sqrt{3} + 1)^2} = a\sqrt{4 + 2\sqrt{3}}$$

**F-3.** A particle simultaneously participates in two mutually perpendicular oscillations  $x = \sin \pi t$  &  $y = 2\cos 2\pi t$ . Write the equation of trajectory of the particle.

दो लम्बवत् दोलन  $x = \sin \pi t$  तथा  $y = 2\cos 2\pi t$  एक साथ एक कण पर कार्यरत है, तो कण के पथ का समीकरण लिखो।

**Ans.**  $2x^2 + \frac{y}{2} = 1$

**Sol.**  $x = \sin \pi t$ ,  $y = 2[1 - 2\sin^2 \pi t]$

$y = 2[1 - 2x^2]$  or  $2x^2 + \frac{y}{2} = 1$

### Section (G) : For JEE-Main के लिए

**G-1.** In forced oscillation of a particle, the amplitude is maximum for a frequency  $\omega_1$  of the force, while the energy is maximum for a frequency  $\omega_2$  of the force. What is the relation between  $\omega_1$  and  $\omega_2$ ?

कण के प्रणोदित दोलन में आयाम बल की  $\omega_1$  आवृत्ति के लिए अधिकतम होता है। जबकि  $\omega_2$  आवृत्ति के लिए ऊर्जा अधिकतम होती है।  $\omega_1$  तथा  $\omega_2$  में सम्बन्ध क्या होगा।

**Ans.** Both amplitude and energy of the particle can be maximum only in the case of resonance. For resonance to occur,  $\omega_1 = \omega_2$

कण का आयाम तथा ऊर्जा दोनों ही अनुनाद की स्थिति में अधिकतम हो सकता है। अनुनाद होने के लिए  $\omega_1 = \omega_2$

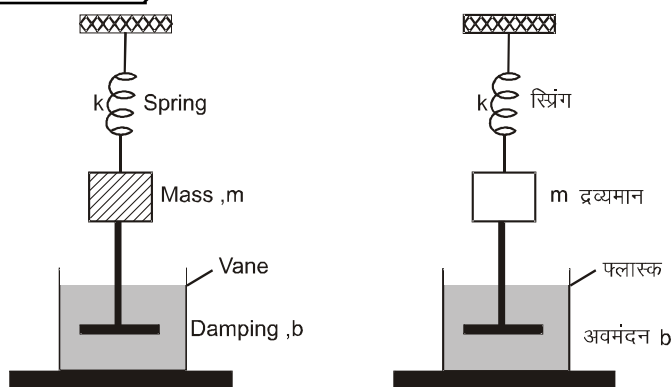
**G-2.#** For the damped oscillator shown in Figure, the mass of the block is 200 g,  $k = 80 \text{ Nm}^{-1}$  and the damping constant  $b$  is  $40 \text{ gs}^{-1}$ . Calculate

चित्र में प्रदर्शित अवमंदित दोलन के लिए ब्लॉक का द्रव्यमान 200 g,  $k = 80 \text{ N m}^{-1}$  है तथा अवमंदन नियतांक  $b = 40 \text{ g s}^{-1}$  है। गणना कीजिए।

(a) The period of oscillation,  
दोलन का आवर्तकाल

(b) Time taken for its amplitude of vibrations to drop to half of its initial value  
दोलन का आयाम प्रारम्भिक मान के आधे होने में लिया गया समय

(c) The time for the mechanical energy to drop to half initial value.  
यांत्रिक ऊर्जा प्रारम्भिक मान की आधी होने में लिया गया समय



**Ans.** (a) 0.3 s (b) 6.93 s (c) 3.4 s

**Sol.** (a) As the damping constant, b

चुंकि अवमंदन नियतांक b

$$(\approx 0.04 \text{ kg s}^{-1}) \ll \sqrt{km}$$

the time period T from the equation

समीकरण से आवर्तकाल T

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \text{ is given by, निम्न प्रकार दिया जाता है}$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.2 \text{ kg}}{80 \text{ Nm}^{-1}}} = 0.314 \text{ s}$$

(b) From the equation  $x(t) = x_m e^{-\frac{bt}{2m}}$  the time  $T_{1/2}$  for the amplitude to drop to half of its initial value is

समीकरण से  $x(t) = x_m e^{-\frac{bt}{2m}}$  आयाम प्रारम्भिक मान के आधे तक होने में लिया गया समय  $T_{1/2}$  होगा।

$$\frac{1}{2} = e^{-\frac{bT_{1/2}}{2m}}$$

$$\text{or या } \log_e 2 = \frac{bT_{1/2}}{2m}$$

$$\text{or या } 0.693 = \frac{bT_{1/2}}{2m}$$

$$\text{or या } T_{1/2} = \frac{0.693 \times 2 \times 0.2}{40 \times 10^{-3}} = 6.93 \text{ s}$$

$$(c) E(t) = E(0) e^{-\frac{bt}{m}};$$

$$\frac{1}{2} = e^{-\frac{bt_{1/2}}{m}}; \log_e 2 = \frac{bt_{1/2}}{m}$$

$$t_{1/2} = \frac{0.693 \times 0.2}{40 \times 10^{-3}} = 3.4 \text{ sec}$$



## PART - II : ONLY ONE OPTION CORRECT TYPE

### भाग - II : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

#### SECTION (A) : EQUATION OF SHM सरल आवर्त गति की समीकरण

- A-1.** According to a scientists, he applied a force  $F = -cx^{1/3}$  on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether  $c$  is a constant or not. Assume that he had worked only with positive  $x$  then:

(A\*) as  $x$  increases  $c$  also increases (B) as  $x$  increases  $c$  decreases  
(C) as  $x$  increases  $c$  remains constant (D) the motion cannot be SHM

वैज्ञानिकों के अनुसार वह एक बल  $F = -cx^{1/3}$  एक कण पर आरोपित करता है तथा कण सरल आवर्त गति करता है। कण पर अन्य कोई बल कार्यरत नहीं है तथा  $c$  नियतांक है या नहीं, यह नहीं बताया जाता है। (माना है कि सिर्फ  $x$  धनात्मक है)

(A\*) जैसाकि  $x$  बढ़ने पर  $c$  बढ़ेगा। (B) जैसाकि  $x$  बढ़ने पर  $c$  घटेगा।  
(C) जैसाकि  $x$  बढ़ने पर  $c$  अपरिवर्तित रहेगा। (D) गति सरल आवर्ती नहीं हो सकती।

- Sol.** Comparing  $F = -kx$   
with  $F = -cx^{1/3}$  के साथ तुलना करने पर  
 $\Rightarrow kx = cx^{1/3} \Rightarrow c = kx^{2/3}$   
As  $x$  increases  $c$  also increases.  $x$  बढ़ने पर  $c$  भी बढ़ता है

- A-2.** A particle performing SHM takes time equal to  $T$  (time period of SHM) in consecutive appearances at a particular point. This point is:

(A\*) An extreme position  
(B) The mean position  
(C) Between positive extreme and mean position  
(D) Between negative extreme and mean position

सरल आवर्त गति करता हुआ एक कण गति के एक निश्चित बिन्दु पर कण की दो क्रमागत उपस्थितियों के मध्य समयान्तराल  $T$  (सरल आवर्त गति का आवर्तकाल) लेता है यह बिन्दु है –

(A\*) सीमान्त स्थिति  
(B) माध्य स्थिति  
(C) धनात्मक सीमान्त स्थिति व माध्य स्थिति के बीच  
(D) ऋणात्मक सीमान्त स्थिति व माध्य स्थिति के बीच

- Sol.** Position where we see the particle once in a time period that is only extreme position. twice through every other position  
वह स्थिति जहाँ कण एक आवर्तकाल में एक बार ही आता है केवल सीमान्त स्थिति है। दूसरी सभी स्थितियों पर दो बार आता है।

- A-3.** A particle executing linear SHM. Its time period is equal to the smallest time interval in which particle acquires a particular velocity  $\vec{v}$ , the magnitude of  $\vec{v}$  may be :

एक कण रेखीय सरल आवर्त गति कर रहा है। कण का आवर्तकाल कण द्वारा विशेष वेग  $\vec{v}$  प्राप्त करने में लिये गये न्यूनतम समयान्तराल के बराबर होता है।  $\vec{v}$  का परिमाण हो सकता है।

(A) Zero शून्य (B\*)  $V_{\max}$  (C)  $\frac{V_{\max}}{2}$  (D)  $\frac{V_{\max}}{\sqrt{2}}$



**Sol.**  $\vec{V}_{\max}$  only केवल  $\vec{V}_{\max}$

If initial velocity is  $\vec{V}_{\max}$

then after one time period particle acquires same speed  $V_{\max}$  in same direction means same velocity

$\vec{V}_{\max}$

यदि प्रारम्भिक वेग  $\vec{V}_{\max}$  हो तो एक आवर्तकाल के पश्चात् कण समान दिशा में समान चाल  $V_{\max}$  प्राप्त कर लेता है अर्थात् समान वेग  $\vec{V}_{\max}$  प्राप्त करता है।

**A-4.** If  $\vec{F}$  is force vector,  $\vec{v}$  is velocity vector,  $\vec{a}$  vector is acceleration vector and  $\vec{r}$  vector is displacement vector with respect to mean position then which of the following quantities are always non-negative in a simple harmonic motion along a straight line?

यदि  $\vec{F}$  बल सदिश  $\vec{v}$  वेग सदिश  $\vec{a}$  त्वरण सदिश तथा  $\vec{r}$  माध्य स्थिति से विस्थापन सदिश है तो एक सरल रेखा के अनुदिश सरल आवर्त गति में निम्न में से कौनसी राशियाँ हमेशा अऋणात्मक रहती है ?

- (A\*)  $\vec{F} \cdot \vec{a}$  (B)  $\vec{v} \cdot \vec{r}$  (C)  $\vec{a} \cdot \vec{r}$  (D)  $\vec{F} \cdot \vec{r}$

**Sol.** (A)  $\vec{F} \cdot \vec{a}$

$m\vec{a} \cdot \vec{a}$  [It is always non-negative] [ये हमेशा अऋणात्मक है]

**A-5.** Two SHM's are represented by  $y = a \sin (\omega t - \phi)$  and  $y = b \cos (\omega t - \phi)$ . The phase difference between the two is :

दो सरल आवर्त गतियों को  $y = a \sin (\omega t - \phi)$  तथा  $y = b \cos (\omega t - \phi)$  से प्रदर्शित करते हैं। इन दोनों के मध्य कलान्तर होगा।

- (A\*)  $\frac{\pi}{2}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{6}$  (D)  $\frac{3\pi}{4}$

**Sol.**  $y = a \sin (\omega t - \phi)$

$$y = b \cos (\omega t - \phi) \Rightarrow y = b \sin (\omega t - \phi + \frac{\pi}{2})$$

So phase difference is  $\pi/2$

अतः कलान्तर  $\pi/2$  है।

**A-6.** How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.

आवर्ती दोलन करते कण की गति प्रारम्भ होने के कितने समय पश्चात् कण का विस्थापन, आयाम का आधा होगा यदि आवर्तकाल 24 सैकण्ड तथा कण स्थिरावस्था से प्रारम्भ होता है।

- (A) 12s (B) 2s (C\*) 4s (D) 6s

**Sol.**  $y = a \cos \omega t$

$$\frac{a}{2} = a \cos \omega t$$

$$\omega t = \frac{\pi}{3}$$

$$\frac{2\pi}{24} t = \frac{\pi}{3}$$

$$t = 4 \text{ sec.}$$



**A-7.** The magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is

सरल आवर्त गति में साम्यावस्था से आधे आवर्त काल में औसत त्वरण का परिमाण होगा—

- (A\*)  $\frac{2A\omega^2}{\pi}$  (B)  $\frac{A\omega^2}{2\pi}$  (C)  $\frac{A\omega^2}{\sqrt{2}\pi}$  (D) Zero शून्य

**Sol.** 
$$a_{\text{avg}} = \frac{\int_0^{T/2} -\omega^2 A \sin \omega t \, dt}{\int_0^{T/2} dt} \Rightarrow \frac{-\omega^2 A \left[ \frac{\cos \omega t}{\omega} \right]_0^{T/2}}{T/2}$$

$$|a|_{\text{avg}} = \frac{-\omega^2 A [-1 - 1]}{T/2} \Rightarrow \frac{4\omega A}{\frac{2\pi}{\omega}} = \frac{2\omega^2 A}{\pi}$$

**A-8.** A particle performing SHM on the y axis according to equation  $y = A + B \sin \omega t$ . Its amplitude is :  
एक कण y-अक्ष पर समीकरण  $y = A + B \sin \omega t$  के अनुसार सरल आवर्त गति करता है, तो इसका आयाम होगा।

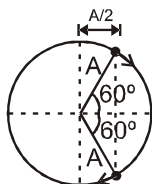
- (A) A (B\*) B (C) A + B (D)  $\sqrt{A^2 + B^2}$

**Sol.**  $X = A + B \sin \omega t$   
 $x - A = B \sin \omega t$   
Hence, Amplitude = B  
अतः आयाम = B

**A-9.** Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement from mean position is half of their amplitudes. The phase-difference between them is दो कण समान सरल रेखा के अनुदिश समान माध्य स्थिति से समान आयाम तथा समान आवृत्ति से सरल आवर्त गति करते हैं। जब वे एक दूसरे को बिना टक्कर के पार (cross) करते हैं, तब वे विपरीत दिशा में गति कर रहे होते हैं और उस क्षण जब माध्य स्थिति से उनका विस्थापन उनके आयाम का आधा होता है, तब उनके मध्य कलान्तर होगा—

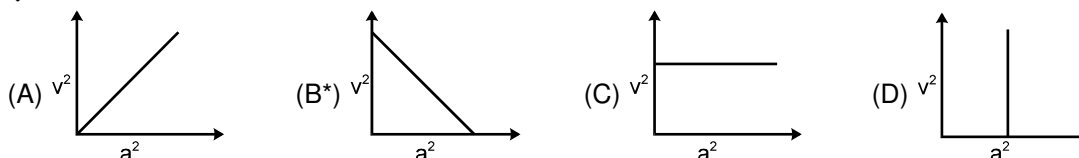
- (A)  $0^\circ$  (B\*)  $120^\circ$  (C)  $180^\circ$  (D)  $135^\circ$

**Sol.** Consider SHM as projection of uniform circular motion.  
From figure the phase difference between two particles is  $120^\circ$ .  
सरल आवर्त को एकसमान वृत्तीय गति का प्रक्षेप मानते हुए  
चित्र से दो कणों के मध्य कलान्तर  $120^\circ$  है।



**A-10.** A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is

एक द्रव्यमान M रेखीय सरल आवर्त गति कर रहा है। त्वरण a तथा संगत रेखीय वेग v के मध्य सही लेखाचित्र होगा—







**Sol.** Velocity वेग  $v = \omega \sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2 \dots(1)$   
 Acceleration त्वरण  $a = -\omega^2 x \Rightarrow a^2 = \omega^4 x^2 \dots(2)$   
 From (1) and (2) :  $v^2 = \omega^2 A^2 - a^2 / \omega^2 \Rightarrow v^2 + a^2 / \omega^2 = \omega^2 A^2$   
 (1) तथा (2) से

$$\Rightarrow \frac{v^2}{\omega^2 A^2} + \frac{a^2}{\omega^4 A^2} = 1$$

$$\Rightarrow v^2 = -a^2 \left( \frac{1}{\omega^2} \right) + 1 \quad \text{its straight line with -ve slope and +ve intercept}$$

ये ऋणात्मक ढाल तथा धनात्मक अन्तःखण्ड वाली सरलरेखा है।

## SECTION (B) : ENERGY ऊर्जा

**B-1.** A body executing SHM passes through its equilibrium. At this instant, it has

सरल आवर्त गति करती वस्तु साम्यावस्था से गुजरती है। इस क्षण, इसमें

- (A) maximum potential energy (B\*) maximum kinetic energy  
 (C) minimum kinetic energy (D) maximum acceleration  
 (A) अधिकतम स्थितिज ऊर्जा होगी। (B\*) अधिकतम गतिज ऊर्जा होगी।  
 (C) न्यूनतम गतिज ऊर्जा होगी। (D) अधिकतम त्वरण होगा।

**Sol.** At equilibrium position K.E. is maximum. साम्यावस्था पर गतिज ऊर्जा अधिकतम होती है।

**B-2.** The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is A आयाम से सरल आवर्त गति करते कण की K.E. तथा P.E. बराबर हो तो इसका विस्थापन होगा

- (A)  $\sqrt{2}A$  (B)  $\frac{A}{2}$  (C\*)  $\frac{A}{\sqrt{2}}$  (D)  $\sqrt{\frac{2}{3}}A$

**Sol.**  $\frac{1}{2} kx^2 = \frac{1}{2} k(A^2 - x^2)$

or या  $x = \frac{A}{\sqrt{2}}$

**B-3.** A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is  $8 \times 10^{-3}$  J. The equation of motion of this particle when the initial phase of oscillation is  $45^\circ$  can be given by

0.1 किग्रा. द्रव्यमान का एक बिन्दु कण 0.1 मीटर के आयाम से सरल आवर्त गति कर रहा है। जब कण अपनी माध्यावस्था से गुजरता है तो इसकी गतिज ऊर्जा  $8 \times 10^{-3}$  जूल है। गति का समीकरण क्या हो सकती है, जबकि दोलन की प्रारम्भिक कला  $45^\circ$  है:

- (A)  $0.1 \cos\left(4t + \frac{\pi}{4}\right)$  (B\*)  $0.1 \sin\left(4t + \frac{\pi}{4}\right)$  (C)  $0.4 \sin\left(t + \frac{\pi}{4}\right)$  (D)  $0.2 \sin\left(\frac{\pi}{2} + 2t\right)$

**Sol.** From question प्रश्न से

$$\frac{1}{2} m\omega^2 A^2 = 8 \times 10^{-3} \Rightarrow \frac{1}{2} \times 0.1 \times \omega^2 \times (0.1)^2 = 8 \times 10^{-3} \Rightarrow \omega = 4$$

So, equation of SHM is अतः स. आ. ग. की समीकरण है  $x = A \sin(\omega t + \phi) = 0.1 \sin\left(4t + \frac{\pi}{4}\right)$ .



**B-4.** For a particle performing SHM :

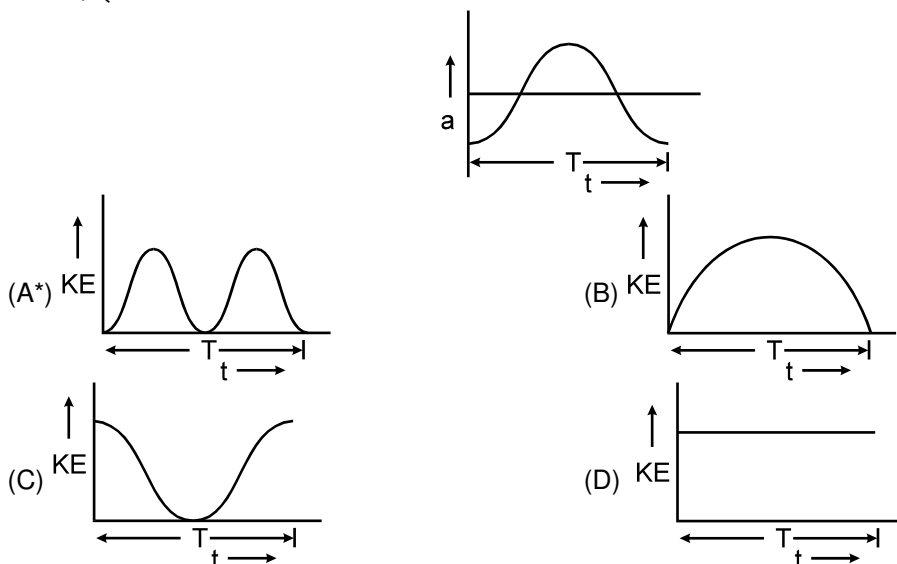
- (A) The kinetic energy is never equal to the potential energy  
 (B) the kinetic energy is always equal to the potential energy  
 (C\*) The average kinetic energy in one time period is equal to the average potential in this period  
 (D) The average kinetic energy in any time interval is equal to average potential energy in that interval

सरल आवर्त गति करते हुए कण के लिए :

- (A) गतिज ऊर्जा कभी भी स्थितिज ऊर्जा के बराबर नहीं होती है।  
 (B) गतिज ऊर्जा हमेशा स्थितिज ऊर्जा के बराबर होती है।  
 (C\*) एक आवर्त काल में औसत गतिज ऊर्जा इस आवर्त काल में औसत स्थितिज ऊर्जा के बराबर होती है।  
 (D) किसी भी समयान्तराल में औसत गतिज ऊर्जा इस समयान्तराल में औसत स्थितिज ऊर्जा के बराबर होती है।

**Sol.**  $P_{AV} = \frac{1}{4} KA^2$  and और  $K_{AV} = \frac{1}{4} KA^2$

**B-5.** Acceleration  $a$  versus time  $t$  graph of a body in SHM is given by a curve shown below.  $T$  is the time period. Then corresponding graph between kinetic energy  $KE$  and time  $t$  is correctly represented by नीचे दिये गये लेखाचित्र में सरल आवर्त गति करती एक वस्तु के त्वरण  $a$  तथा आवर्तकाल  $T$  के मध्य संबंध प्रदर्शित किया गया है। इसके संगत गतिज ऊर्जा  $KE$  व समय  $t$  के मध्य सही लेखाचित्र होगा—



**Sol.**  $x = A \cos \omega t$

$$K.E. = \frac{1}{2} k(A^2 - x^2) = \frac{1}{2} kA^2 \sin^2 \omega t$$

$$= \frac{1}{2} kA^2 \frac{(1 - \cos 2\omega t)}{2}$$

$$= \frac{kA^2}{4} (1 - \cos 2\omega t)$$

Frequency of K.E. is double of acceleration.

गतिज ऊर्जा की आवृत्ति त्वरण की दुगुनी है।





- B-6.** A particle performs S.H.M. of amplitude  $A$  along a straight line. When it is at a distance  $\frac{\sqrt{3}}{2}A$  from mean position, its kinetic energy gets increased by an amount  $\frac{1}{2}m\omega^2 A^2$  due to an impulsive force. Then its new amplitude becomes:

एक कण  $A$  आयाम से सरल रेखा के अनुदिश सरल आवर्त गति करता है जब यह माध्य स्थिति से  $\frac{\sqrt{3}}{2}A$  दूरी पर है तब इसकी गतिज ऊर्जा, आवेगीय बल द्वारा  $\frac{1}{2}m\omega^2 A^2$  बढ़ा दी जाती है। अतः इसका नया आयाम है :

- (A)  $\frac{\sqrt{5}}{2}A$  (B)  $\frac{\sqrt{3}}{2}A$  (C\*)  $\sqrt{2}A$  (D)  $\sqrt{5}A$

**Sol.** Due to impulse force, the total energy of the particle becomes :

$$\frac{1}{2}m\omega^2 A^2 + \frac{1}{2}m\omega^2 A^2 = m\omega^2 A^2$$

Let ;  $A'$  be the new amplitude. (Apply energy conservation law)

$$\therefore \frac{1}{2}m\omega^2 (A')^2 = m\omega^2 A^2 \Rightarrow A' = \sqrt{2}A. \quad \text{Ans.}$$

**Sol.** आवेगीय बल के कारण, कण की कुल ऊर्जा :

$$\frac{1}{2}m\omega^2 A^2 + \frac{1}{2}m\omega^2 A^2 = m\omega^2 A^2$$

माना  $A'$  नया आयाम है। (ऊर्जा संरक्षण के नियम से)

$$\therefore \frac{1}{2}m\omega^2 (A')^2 = m\omega^2 A^2 \Rightarrow A' = \sqrt{2}A. \quad \text{Ans.}$$

### SECTION (C) : SPRING MASS SYSTEM स्प्रिंग द्रव्यमान निकाय

- C-1.** Two spring mass systems have equal mass and spring constant  $k_1$  and  $k_2$ . If the maximum velocities in two systems are equal then ratio of amplitude of 1st to that of 2nd is :
- समान द्रव्यमान तथा स्प्रिंग नियतांक क्रमशः  $k_1$  व  $k_2$  के दो स्प्रिंग द्रव्यमान निकाय हैं। यदि इनके अधिकतम वेग समान हैं तो 1st के आयाम का 2nd के आयाम के साथ अनुपात है—

- (A)  $\sqrt{k_1/k_2}$  (B)  $k_1/k_2$  (C)  $k_2/k_1$  (D\*)  $\sqrt{k_2/k_1}$

**Sol.**  $\frac{1}{2}mv^2 = \frac{1}{2}K_1x_1^2$

$$\frac{1}{2}mv^2 = \frac{1}{2}K_2x_2^2$$

$$K_1x_1^2 = K_2x_2^2$$

$$\frac{x_1}{x_2} = \sqrt{\frac{K_2}{K_1}}$$

- C-2.#** A toy car of mass  $m$  is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is  $k$  and surface is frictionless. The car is displaced from mean position by  $x$  cm and released. At the mean position the ribbons are undeformed. Vibration period is
- एक  $m$  द्रव्यमान की खिलौना गाड़ी से दो समान रबर के रिबन चित्र में दिखाए अनुसार बांधे गये हैं। रबर के रिबन का बल नियतांक  $k$  तथा सतह घर्षणहीन है। साम्यावस्था पर रिबन अविरूपित अवस्था में हैं। कार को माध्यावस्था से  $x$  cm विस्थापित करके छोड़ दिया जाए तो कम्पन्न काल होगा।

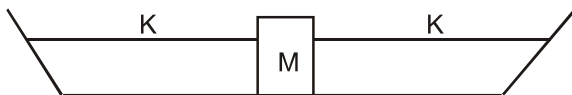


- (A)  $2\pi\sqrt{\frac{m(2k)}{k^2}}$  (B)  $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$  (C\*)  $2\pi\sqrt{\frac{m}{k}}$  (D)  $2\pi\sqrt{\frac{m}{k+k}}$





Sol.



Rubber ribbon can exert only tension not compression so at a time only one is effective.

रबर रिबन केवल तनाव आरोपित कर सकता है, सम्पीडन नहीं अतः एक समय में केवल एक ही प्रभावी रहेगा।

$$T = 2\pi \sqrt{\frac{m}{K}}$$

- C-3.** A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half :

एक 1 किग्रा. का द्रव्यमान स्प्रिंग के निचले सिरे से जुड़ा है तथा किसी निश्चित आवृत्ति से कम्पन कर रहा है। आवृत्ति के इस निश्चित मान को आधा करने के लिए निम्न द्रव्यमान जोड़ना पड़ेगा :

- (A) 1 kg (B) 2 kg (C\*) 3 kg (D) 4 kg

Sol.

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{K}{m_1}}$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{K}{m_2}}$$

$$f_2 = \frac{f_1}{2} \text{ or } m_2 = 4m_1 \text{ or या } m_2 - m_1 = 3 \text{ kg}$$

- C-4.** A ball of mass  $m$  kg hangs from a spring of spring constant  $k$ . The ball oscillates with a period of  $T$  seconds. If the ball is removed, the spring is shortened (with respect to length in mean position) by  $m$  किग्रा द्रव्यमान की एक गेंद,  $k$  स्प्रिंग नियतांक वाली स्प्रिंग से लटकायी जाती है। गेंद  $T$  सैकण्ड आवर्त काल से दोलन करती है। यदि गेंद को हटा दिया जाए तो स्प्रिंग निम्न लम्बाई से छोटी (माध्य स्थिति से लम्बाई के सापेक्ष) हो जायेगी –

- (A\*)  $\frac{gT^2}{(2\pi)^2}$  metre मीटर (B)  $\frac{3T^2g}{(2\pi)^2}$  metre मीटर (C)  $\frac{Tm}{k}$  metre मीटर (D)  $\frac{Tk}{m}$  metre मीटर

Sol.

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$m = \frac{T^2}{4\pi^2} k$$

$$mg = Kx$$

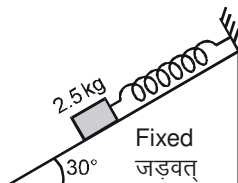
$$x = \frac{mg}{K}$$

$$x = \frac{T^2 K}{4\pi^2} \frac{g}{K}$$

$$x = \frac{T^2 g}{4\pi^2}$$

- C-5.** A smooth inclined plane having angle of inclination  $30^\circ$  with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is

दिये गये चित्र में एक चिकने नत तल का क्षैतिज के साथ कोण  $30^\circ$  है। इस नत तल पर 2.5 kg द्रव्यमान का एक गुटका रखा हुआ है जिसे स्प्रिंग के एक सिरे से जोड़ा हुआ है। स्प्रिंग का दूसरा सिरा नत तल के ऊपरी सिरे से जुड़ा है। यदि द्रव्यमान को नत तल के अनुदिश 2.5 सेमी ऊपर ले जाया जाता है तो स्प्रिंग में तनाव घटकर शून्य हो जाता है। यदि अब द्रव्यमान को स्वतंत्र छोड़ दिया जाए तो दोलनों की कोणीय आवृत्ति रेडियन/सेकण्ड में होगी :-



- (A) 0.707

- (B) 7.07

- (C) 1.414

- (D\*) 14.14



**Sol.**  $kx = mg \sin 30^\circ$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g \sin 30^\circ}{x}} = \sqrt{\frac{5 \times 100}{2.5}} = 14.14 \quad \text{Ans.}$$

**C-6.** A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is  $T$ . If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

स्प्रिंग द्वारा आरोपित प्रत्यानयन बल के कारण एक कण सरल आवर्त गति करता है। आवर्त काल  $T$  है। यदि स्प्रिंग को दो भागों में विभाजित कर दिया जाये तथा इसके आधे एक भाग से सरल आवर्त गति कराई जाय तो आवर्त काल –

- (A) remain  $T$  (B) become  $2T$  (C) become  $T/2$  (D\*) become  $T/\sqrt{2}$   
 (A)  $T$  के बराबर होगा। (B)  $2T$  हो जाएगा। (C)  $T/2$  हो जाएगा। (D\*)  $T/\sqrt{2}$  हो जाएगा।

**Sol.** Time period आवर्तकाल  $= T = 2\pi \sqrt{\frac{m}{K}}$

Spring divided into two equal parts so length is reduced to half  
 स्प्रिंग दो बराबर भागों में विभाजित की गयी है अतः लम्बाई आधी हो जायेगी

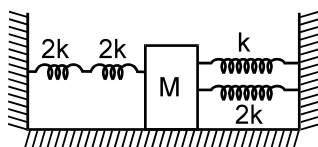
We know हम जानते हैं  $K \propto \frac{1}{\ell}$

$\therefore$   $K$  become twice  $K$  दुगुना हो जायेगा।

$$T_{\text{new}} = 2\pi \sqrt{\frac{m}{K_{\text{new}}}} = 2\pi \sqrt{\frac{m}{2K}} = \frac{1}{\sqrt{2}} \left( 2\pi \sqrt{\frac{m}{K}} \right) = \frac{T}{\sqrt{2}}$$

**C-7.** Four massless springs whose force constants are  $2k$ ,  $2k$ ,  $k$  and  $2k$  respectively are attached to a mass  $M$  kept on a frictionless plane (as shown in figure). If the mass  $M$  is displaced in the horizontal direction, then the frequency of the system.

चित्र में दिखाये अनुसार घर्षण रहित तल पर रखे हुए  $M$  द्रव्यमान के एक ब्लॉक से चार स्प्रिंग जुड़ी हुई है। जिनके बल नियतांक  $2k$ ,  $2k$ ,  $k$  तथा  $2k$  है। यदि  $M$  द्रव्यमान को क्षैतिज दिशा में विस्थापित किया जाए तो निकाय की आवृत्ति होगी :



- (A)  $\frac{1}{2\pi} \sqrt{\frac{k}{4M}}$  (B\*)  $\frac{1}{2\pi} \sqrt{\frac{4k}{M}}$  (C)  $\frac{1}{2\pi} \sqrt{\frac{k}{7M}}$  (D)  $\frac{1}{2\pi} \sqrt{\frac{7k}{M}}$

**Sol.**  $k_{\text{eq}} = 2k + k + \frac{2k \times 2k}{2k + 2k} = 4k$

so, frequency, अतः आवृत्ति  $f = \frac{1}{2\pi} \sqrt{\frac{K_{\text{eq}}}{M}} = \frac{1}{2\pi} \sqrt{\frac{4K}{M}}$

**C-8.** The total mechanical energy of a particle of mass  $m$  executing SHM with the help of a spring is  $E = (1/2)m\omega^2 A^2$ . If the particle is replaced by another particle of mass  $m/2$  while the amplitude  $A$  remains same. New mechanical energy will be :

एक स्प्रिंग की मदद से सरल आवर्त गति करते हुए  $m$  द्रव्यमान के एक कण की कुल यांत्रिक ऊर्जा  $E = \frac{1}{2} m \omega^2 A^2$  है।

माना कण को दूसरे  $m/2$  द्रव्यमान के कण से प्रतिस्थापित कर दिया जाता है जबकि आयाम  $A$  अपरिवर्तित है तो नई यांत्रिक ऊर्जा होगी।

- (A)  $\sqrt{2} E$  (B)  $2E$  (D)  $E/2$  (D\*)  $E$



**Sol.**  $E = \frac{1}{2} m \omega^2 A^2$   
 $= \frac{1}{2} m \times \frac{k}{m} \times A^2$   
 $= \frac{1}{2} k A^2$

E is independent of mass.

E द्रव्यमान पर निर्भर नहीं करती है।

## SECTION (D) : SIMPLE PENDULUM सरल लोलक

**D-1.** Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.

दो लोलक एक साथ दोलन प्रारम्भ करते हैं। पहला लोलक 9 दोलन पूरे करता है, जब दूसरा 7 पूरे करता है। दोनों लोलकों की लम्बाईयों का अनुपात होगा।

- (A\*)  $\frac{49}{81}$  (B)  $\frac{7}{9}$  (C)  $\frac{50}{81}$  (D)  $\frac{1}{2}$

**Sol.** Given time for both are same दिया समय दोनों के लिये समान है।

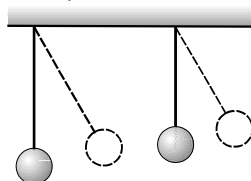
$$9T_1 = 7T_2$$

$$9 \times 2\pi \sqrt{\frac{\ell_1}{g}} = 7 \times 2\pi \sqrt{\frac{\ell_2}{g}}$$

$$\Rightarrow 9\sqrt{\ell_1} = 7\sqrt{\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{49}{81}$$

**D-2.#** Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) :

दो लोलक जो कि प्रारम्भ में स्थिरावस्था में हैं, एक साथ दोलन प्रारम्भ करते हैं। उनकी लम्बाई क्रमशः 1.44 मी. तथा 1 मी. है। वे पुनः एक साथ एक ही कला में होंगे (लम्बे लोलक के सापेक्ष) –



- (A) 1 vibration (B) 3 vibrations (C) 4 vibrations (D\*) 5 vibrations  
 (A) 1 कम्पन् के बाद (B) 3 कम्पन् के बाद (C) 4 कम्पन् के बाद (D\*) 5 कम्पन् के बाद

**Sol.** Let माना  $x_1 = A_1 \sin \omega_1 t$  and  $x_2 = A_2 \sin \omega_2 t$

Two pendulums will vibrate in same phase again when there phase difference  $(\omega_2 - \omega_1)t = 2\pi$   
 दो लोलक दुबारा समान कला में कम्पन् करेंगे जब उनका कलान्तर  $(\omega_2 - \omega_1)t = 2\pi$  होगा।

$$\Rightarrow \left( \frac{2\pi}{T_2} - \frac{2\pi}{T_1} \right) t = 2\pi$$

$$\Rightarrow \left( \sqrt{\frac{g}{1}} - \sqrt{\frac{g}{1.44}} \right) n \times T_1 = 2\pi \text{ (where } n \text{ is number of vibrations completed by longer pendulum)}$$

(जहाँ n लम्बे लोलक द्वारा पूरे किये गये दोलनों की संख्या है)

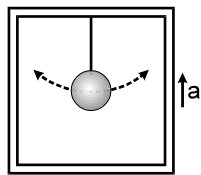
$$\Rightarrow \left( \sqrt{\frac{g}{1}} - \sqrt{\frac{g}{1.44}} \right) n \times 2\pi \sqrt{\frac{1.44}{g}} = 2\pi \Rightarrow n = 5$$

Thus after 5 vibrations of longer pendulum they will again start swinging in same phase.

अतः बड़े लोलक के 5 कम्पन् के बाद ये दुबारा समान कला में दोलन प्रारम्भ करेंगे।



- D-3.#** A scientist measures the time period of a simple pendulum as  $T$  in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is  
 एक वैज्ञानिक एक स्थिर लिफ्ट में सरल लोलक का आवर्तकाल  $T$  मापता है। यदि लिफ्ट ऊपर की तरफ  $g/4$  त्वरण से गति करे तो नया आवर्तकाल होगा:-



- (A)  $\frac{T}{4}$  (B)  $4T$  (C\*)  $\frac{2}{\sqrt{5}}T$  (D)  $\frac{\sqrt{5}}{2}T$

**Sol.**  $T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{\ell}{g + g/4}} = \frac{2}{\sqrt{5}}T$

- D-4.** A simple pendulum has some time period  $T$ . What will be the percentage change in its time period if its amplitude is decreased by 5%?

एक सरल लोलक का आवर्तकाल  $T$  है। यदि इसका आयाम 5% कम हो जाये तो इसके आवर्तकाल में कितने प्रतिशत परिवर्तन हो जाएगा ?

- (A) 6 % (B) 3 % (C) 1.5 % (D\*) 0 %

**Sol.**  $T = 2\pi\sqrt{\frac{\ell}{g}}$ , As it does not depend on amplitude चूंकि ये आयाम पर निर्भर नहीं करता है

∴ % change in time period is 0 % Hence option (D) is correct.  
 आवर्त काल में प्रतिशत परिवर्तन 0 % है अतः विकल्प (D) सही है।

- D-5.** A simple pendulum with length  $\ell$  and bob of mass  $m$  executes SHM of small amplitude  $A$ . The maximum tension in the string will be

एक सरल लोलक जिसकी लम्बाई  $\ell$  व गोलक का द्रव्यमान  $m$  है,  $A$  लघु आयाम से सरल आवर्त गति करता है। रस्सी में अधिकतम तनाव होगा –

- (A)  $mg(1 + A/\ell)$  (B)  $mg(1 + A/\ell)^2$  (C\*)  $mg[1 + (A/\ell)^2]$  (D)  $2mg$

**Sol.**  $T_{\text{max}} = mg + m\omega^2\ell = mg + m\frac{v^2}{\ell} = mg + m\frac{(\omega A)^2}{\ell} = mg + m\frac{g}{\ell} \times \frac{A^2}{\ell} \left( \because \omega = \sqrt{\frac{g}{\ell}} \right)$

or या  $T_{\text{max}} = mg + mg\left(\frac{A}{\ell}\right)^2 = mg\left[1 + \left(\frac{A}{\ell}\right)^2\right]$

## SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

### पिण्ड लोलक व मरोड़ी लोलक

- E-1.** A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then the orientation is maintained. If the sphere is then released, its time period of the oscillation will be :

एक 25 किग्रा. द्रव्यमान का ठोस गोला जिसकी त्रिज्या 20 सेमी. है, एक उर्ध्व तार से इस प्रकार लटकाया जाता है कि निलम्बन बिन्दु गोले के केन्द्र से ठीक ऊपर (उर्ध्वाधर) है। गोले को 1.0 रेडियन कोण से घुमाने के लिए तथा उसके बाद अपनी स्थिति को बनाये रखने के लिए 0.10 न्यूटन-मी. का बल-आघूर्ण आवश्यक है। अगर तत्पश्चात् गोले को मुक्त छोड़ दिया जाये तो इसके दोलों का आवर्तकाल होगा –

- (A)  $\pi$  second (B)  $\sqrt{2}\pi$  second (C)  $2\pi$  second (D\*)  $4\pi$  second  
 (A)  $\pi$  सैकण्ड (B)  $\sqrt{2}\pi$  सैकण्ड (C)  $2\pi$  सैकण्ड (D\*)  $4\pi$  सैकण्ड





**Sol.**  $I = \frac{2}{5} mR^2 = \frac{2}{5} \times 25 \times (0.2)^2 = \frac{2}{5}$

$\tau = C\theta$   $c = \frac{\tau}{\theta} = \frac{0.1}{1} = 0.1$

$T = 2\pi \sqrt{\frac{I}{C}} = 2\pi \sqrt{\frac{2}{5 \times 0.1}} = 2\pi \times 2 = 4\pi \text{ secs}$

**E-2.** A metre stick swinging about its one end oscillates with frequency  $f_0$ . If the bottom half of the stick was cut off, then its new oscillation frequency will be:

एक सिरे के सापेक्ष झूल रही एक मीटर छड़  $f_0$  आवृत्ति से दोलन करती है। अगर छड़ का नीचे का आधा भाग काट दिया जाए तो दोलनों की नयी आवृत्ति क्या होगी –

- (A)  $f_0$  (B\*)  $\sqrt{2} f_0$  (C)  $2f_0$  (D)  $2 f_0$

**Sol.**  $f_0 = \frac{1}{2\pi} \sqrt{\frac{mg\ell}{I}}$

where,  $\ell$  is distance between point of suspension and centre of mass of the body.

Thus, for the stick of length  $\ell$  and mass  $m$  :

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{m \cdot g \cdot \frac{\ell}{2}}{(mL^2/3)}} = \frac{1}{2\pi} \sqrt{\frac{3g}{2\ell}}$$

when bottom half of the stick is cut of

$$f_0' = \frac{1}{2\pi} \sqrt{\frac{\frac{m}{2} \cdot g \cdot \frac{\ell}{4}}{\frac{m(\ell/2)^2}{3}}} = \frac{1}{2\pi} \times \sqrt{\frac{3g}{\ell}} = \sqrt{2} f_0 \quad \text{Ans.}$$

**Sol.**  $f_0 = \frac{1}{2\pi} \sqrt{\frac{mg\ell}{I}}$

यहाँ,  $\ell$  निलम्बन बिन्दु व वस्तु के द्रव्यमान केन्द्र के बीच की दूरी है।

अतः द्रव्यमान  $m$  तथा  $\ell$  लम्बाई की छड़ के लिए

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{m \cdot g \cdot \frac{\ell}{2}}{(mL^2/3)}} = \frac{1}{2\pi} \sqrt{\frac{3g}{2\ell}}$$

जब छड़ को आधा काट लें तो

$$f_0' = \frac{1}{2\pi} \sqrt{\frac{\frac{m}{2} \cdot g \cdot \frac{\ell}{4}}{\frac{m(\ell/2)^2}{3}}} = \frac{1}{2\pi} \times \sqrt{\frac{3g}{\ell}} = \sqrt{2} f_0 \quad \text{Ans.}$$

## SECTION (F) : SUPERPOSITION OF SHM सरल आवर्त गति का अध्यारोपण

**F-1.** When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

जब समान आवृत्ति, समान आयाम तथा समान कला की दो परस्पर लम्बवत् सरल आवर्त गतियाँ अध्यारोपित होती हैं तो –

(A) the resulting motion is uniform circular motion.

परिणामी गति समरूप वृत्तीय गति होती है।

(B\*) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.

परिणामी गति, गतियों की सरल रेखाओं के एक घटक से समान रूप से झुकी हुई सरल रेखा के अनुदिश रेखीय सरल आवर्त गति है।

(C) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.

परिणामी गति, गति के घटकों की रेखा के सापेक्ष सममित दीर्घवृत्तीय गति होगी।

(D) the two S.H.M. will cancel each other.

दोनों सरल आवर्त गति एक दूसरे को निरस्त कर देंगी।



**Sol.**  $y = A \sin(\omega t + \phi)$  and तथा  $x = A \sin(\omega t + \phi)$   
then तो  $y = x$  so path is straight line. अतः पथ सरल रेखा है

**F-2.** The position of a particle in motion is given by  $y = C \sin \omega t + D \cos \omega t$  w.r.t. origin. Then motion of the particle is:

गति में एक कण की मूल बिन्दु के सापेक्ष स्थिति  $y = C \sin \omega t + D \cos \omega t$  से दी जाती है तो कण की गति है

(A) SHM with amplitude  $C+D$

(B\*) SHM with amplitude  $\sqrt{C^2 + D^2}$

(C) SHM with amplitude  $\frac{(C+D)}{2}$

(D) not SHM

(A)  $C + D$  आयाम की सरल आवर्त गति है।

(B\*)  $\sqrt{C^2 + D^2}$  आयाम की सरल आवर्त गति है।

(C)  $(C + D)/2$  आयाम की सरल आवर्त गति है।

(D) सरल आवर्त गति नहीं है।

**Sol.**  $x = C \sin \omega t + D \sin(\omega t + \pi/2)$

$$A_r = \sqrt{C^2 + D^2 + 2CD \cos \frac{\pi}{2}} \quad A_r = \sqrt{C^2 + D^2}$$

**F-3.** A simple harmonic motion is given by  $y = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$ . What is the amplitude of motion if  $y$  is in m?

एक सरल आवर्त गति  $y = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$  से प्रदर्शित की जाती है। गति का आयाम क्या होगा, यदि  $y$  मीटर में है?

(A) 100 cm

(B) 5 m

(C) 200 cm

(D\*) 1000 cm

**Sol.**  $y = 10 \left( \frac{1}{2} \sin 3\pi t + \frac{\sqrt{3}}{2} \cos 3\pi t \right) = 10 \sin \left( 3\pi t + \frac{\pi}{3} \right)$

thus amplitude is अतः आयाम 10 m or या 1000 cm

**F-4.** The position vector of a particle moving in x-y plane is given by

$\vec{r} = (A \sin \omega t) \hat{i} + (A \cos \omega t) \hat{j}$  then motion of the particle is :

(A) SHM

(B\*) on a circle

(C) on a straight line

(D) with constant acceleration

x-y तल में गति करते हुए एक कण का स्थिति सदिश  $\vec{r} = (A \sin \omega t) \hat{i} + (A \cos \omega t) \hat{j}$  से दिया जाता है तो कण की गति

(A) सरल आवर्त गति है।

(B\*) वृत्त पर गति है।

(C) सरल रेखा पर गति है।

(D) नियत त्वरण से होती है।

**Sol.**  $x = A \sin \omega t, y = A \cos \omega t$  or या  $x^2 + y^2 = A^2$

Thus the motion of the particle is on a circle. अतः कण की गति वृत्त पर है।

## SECTION (G) : FOR JEE MAIN

**G-1.** When an oscillator completes 100 oscillations its amplitude reduced to  $\frac{1}{3}$  of initial value. What will be its amplitude, when it completes 200 oscillations :

जब एक अवमन्दित दोलक 100 दोलन पूरे करता है, तो उसका आयाम प्रारम्भिक मान का  $\frac{1}{3}$  हो जाता है। 200 दोलन पूरे करने के पश्चात् इसका आयाम हो जायेगा :

(1)  $\frac{1}{8}$

(2)  $\frac{2}{3}$

(3)  $\frac{1}{6}$

(4\*)  $\frac{1}{9}$



**Sol.** In case of damped vibration, amplitude at any instant is  
अवमन्दित कम्पन्न की स्थिति में किसी क्षण पर आयाम है।

$$a = a_0 e^{-bt}$$

where  $a_0$  = initial amplitude

जहाँ  $a_0$  = प्रारम्भिक आयाम

$b$  = damping constant अवमन्दन गुणांक

**Ist caes :**  $t = 100 T$  and तथा  $a = \frac{a_0}{3}$

$$\therefore \frac{a_0}{3} = a_0 e^{-b(100 T)}$$

$$\Rightarrow e^{-100 bT} = \frac{1}{3}$$

**IInd caes :**  $t = 200 T$   
 $a = a_0 e^{-bt} = a_0 e^{-b(200 T)}$

$$= a_0 (e^{-100 bT})^2 = a_0 \times \left(\frac{1}{3}\right)^2 = \frac{a_0}{9}$$

**G-2.** The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are:

किसी दोलित्र पर अवमन्दक-बल वेग के समानुपाती होता है तो, समानुपाती नियतांक का मात्रक है :

- (1)  $\text{kgms}^{-1}$  (2)  $\text{kgms}^{-2}$  (3\*)  $\text{kgs}^{-1}$  (4)  $\text{kg}$

**Sol.**  $F \propto v \Rightarrow F = kV$

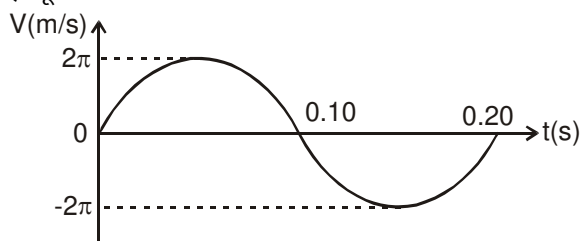
$$k = \frac{F}{v} \Rightarrow [k] = \frac{[\text{kgms}^{-2}]}{[\text{ms}^{-1}]} = \text{kg s}^{-1}$$

## PART - III : MATCH THE COLUMN

### भाग - III : कॉलम को सुमेलित कीजिए (MATCH THE COLUMN)

**1. #** A simple harmonic oscillator consists of a block attached to a spring with  $k = 200 \text{ N/m}$ . The block slides on a frictionless horizontal surface, with equilibrium point  $x = 0$ . A graph of the block's velocity  $v$  as a function of time  $t$  is shown. Correctly match the required information in the left column with the values given in the right column. (use  $\pi^2 = 10$ )

एक सरल आवर्त गति कम्पित्र में,  $k = 200 \text{ N/m}$  की स्प्रिंग के साथ एक पिण्ड जुड़ा है। पिण्ड घर्षण रहित क्षैतिज सतह पर साम्यवस्था बिन्दु  $x = 0$  के साथ सरकता है। पिण्ड के वेग  $v$  का समय  $t$  के फलन के रूप में चित्रण ग्राफ में दर्शाया गया है। बायें स्तम्भ में दी गई सूचना को दायें स्तम्भ में दिये गये परिमाणों से सही मिलाओ – ( $\pi^2 = 10$  का प्रयोग करें)



Left Column	Right Column
(A) The block's mass in kg	(p) $-0.20$
(B) The block's displacement at $t = 0$ in metres	(q) $-200$
(C) The block's acceleration at $t = 0.10 \text{ s}$ in $\text{m/s}^2$	(r) $0.20$
(D) The block's maximum kinetic energy in Joule	(s) $4.0$



## बायाँ स्तम्भ

- (A) पिण्ड का द्रव्यमान किग्रा. में।  
 (B)  $t = 0$  पर पिण्ड का विस्थापन मीटर में।  
 (C)  $t = 0.10$  सेकण्ड पर पिण्ड का त्वरण मी./से.<sup>2</sup> में।  
 (D) पिण्ड की महत्तम गतिज ऊर्जा जूल में।

## दायाँ स्तम्भ

- (p) – 0.20  
 (q) – 200  
 (r) 0.20  
 (s) 4.0

**Ans.** (A) r, (B) p, (C) q, (D) s

**Sol.**  $V_{\max} = A\omega$

$$\Rightarrow A = \frac{V_{\max}}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20 \text{ m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2 \text{ kg}$$

At  $t = 0.1$ , acc. is maximum

$t = 0.1$  पर त्वरण महत्तम है –

$$\Rightarrow a_{\max} = -\omega^2 A = -\left(\frac{2\pi}{0.2}\right)^2 \times 0.2$$

$$= -200 \text{ m/s}^2$$

$$\text{Maximum energy} = \frac{1}{2} m V_{\max}^2 = 4 \text{ J}$$

$$\text{महत्तम ऊर्जा} = \frac{1}{2} m V_{\max}^2 = 4 \text{ J}$$

**2.** In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

स्तम्भ-I में एक निकाय की स्थिति को प्रत्येक विकल्प बताया गया है तथा उसके संगत आवर्तकाल स्तम्भ-II में दिया गया है। तो इनको सुमेलित करो।

## Column-I

- (A) A simple pendulum of length ' $\ell$ ' oscillating with small amplitude in a lift moving down with retardation  $g/2$ .
- (B) A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a stationary lift, stretches the spring by length ' $\ell$ ' in equilibrium. It's time period when lift moves up with an acceleration  $g/2$  is
- (C) The time period of small oscillation of a uniform rod of length ' $\ell$ ' smoothly hinged at one end. The rod oscillates in vertical plane.
- (D) A cubical block of edge ' $\ell$ ' and specific gravity  $1/2$  is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is

## Column-II

(p)  $T = 2\pi\sqrt{\frac{2\ell}{3g}}$

(q)  $T = 2\pi\sqrt{\frac{\ell}{g}}$

(r)  $T = 2\pi\sqrt{\frac{2\ell}{g}}$

(s)  $T = 2\pi\sqrt{\frac{\ell}{2g}}$



## स्तम्भ-I

## स्तम्भ-II

- (A) एक सरल लोलक जो कि लम्बाई ' $\ell$ ' का है जो कि एक अल्प आयाम के साथ लिफ्ट में दोलन कर रहा है। लिफ्ट नीचे की तरफ मन्दन  $g/2$  से जा रही है।
- (B) एक ब्लॉक को एक ऊर्ध्वाधर स्प्रिंग के एक सिरे पर लगा दिया जाता है जिसका दूसरा सिरा एक स्थिर लिफ्ट की छत से जड़वत लगा है तथा साम्यवस्था में यह स्प्रिंग में  $\ell$  विस्तार करती है। इसका आवर्तकाल क्या होगा जब लिफ्ट ऊपर की तरफ  $g/2$  त्वरण से जा रही होगी।
- (C)  $\ell$  लम्बाई की एक समरूप छड़ को एक सिरे पर बिना घर्षण के किलकीत कर दिया जाता है। यह छड़ ऊर्ध्वाधर तल में दोलन कर रही है। तो छोटे दोलनों के लिए इसका आवर्तकाल होगा।
- (D) एक घनाकार ब्लॉक की भुजा  $\ell$  है तथा विशिष्ट घनत्व (गुरुत्व)  $1/2$  है तथा इसका कुछ आयतन पानी के अन्दर है जो कि एक बड़े जड़वत पात्र में भरा हुआ है, यह साम्यवस्था में है। श्यान बलों व पृष्ठतनाव को नगण्य मानो तो ऊर्ध्वाधर दिशा में छोटे दोलनों के लिए ब्लॉक का आवर्तकाल होगा।

$$(p) T = 2\pi \sqrt{\frac{2\ell}{3g}}$$

$$(q) T = 2\pi \sqrt{\frac{\ell}{g}}$$

$$(r) T = 2\pi \sqrt{\frac{2\ell}{g}}$$

$$(s) T = 2\pi \sqrt{\frac{\ell}{2g}}$$

**Ans.** (A) p (B) q (C) p (D) s

**Sol.** (A) In frame of lift effective acceleration due to gravity is  $g + \frac{g}{2} = \frac{3g}{2}$  downwards

लिफ्ट के फ्रेम में गुरुत्व के कारण त्वरण  $= g + \frac{g}{2} = \frac{3g}{2}$  नीचे की तरफ

$$\therefore T = 2\pi \sqrt{\frac{2\ell}{3g}}$$

$$(B) K\ell = mg \quad \therefore \frac{k}{m} = \frac{g}{L}$$

constant acceleration of lift has no effect in time period of oscillation.

लिफ्ट का नियत त्वरण, दोलन के आवर्तकाल पर कोई असर नहीं डालता है।

$$\therefore T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{\ell}{g}}$$

$$(C) T = 2\pi \sqrt{\frac{I}{mgd}} = 2\pi \sqrt{\frac{\frac{m\ell^2}{3}}{mg \frac{\ell}{2}}} = 2\pi \sqrt{\frac{2\ell}{3g}}$$

$$(D) T = 2\pi \sqrt{\frac{m}{\rho Ag}} = 2\pi \sqrt{\frac{(\rho/2) A\ell}{\rho Ag}} = 2\pi \sqrt{\frac{\ell}{2g}}$$



## Exercise-2

Marked Questions may have for Revision Questions.

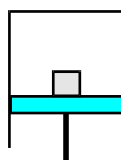
चिह्नित प्रश्न दोहराने योग्य प्रश्न है।

### PART - I : ONLY ONE OPTION CORRECT TYPE

भाग-I : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

#### SECTION (A) : EQUATION OF SHM सरल आवर्त गति का समीकरण

- 1.# A block of mass  $m$  is resting on a piston as shown in figure which is moving vertically with a SHM of period  $1$  s. The minimum amplitude of motion at which the block and piston separate is :  
 दिये गये चित्र में एक गुटका जिसका द्रव्यमान  $m$  है, एक पिस्टन पर स्थिरावस्था में है। पिस्टन  $1$  sec. के आवर्तकाल से ऊर्ध्वाधर सरल आवर्त गति कर रहा है। गति का वह न्यूनतम आयाम क्या होगा, जिस पर गुटका तथा पिस्टन एक दूसरे से अलग हो जायें –



(A\*)  $0.25$  m

(B)  $0.52$  m

(C)  $2.5$  m

(D)  $0.15$  m

Sol. Acceleration त्वरण  $(a) = \omega^2 x \Rightarrow a_{\max} = \left(\frac{2\pi}{T}\right)^2 A$

When  $a_{\max} = g$  then block and piston will be separated

जब  $a_{\max} = g$  वस्तु तथा पिस्टन अलग हो जायेंगे।

$$a_{\max} = g = \left(\frac{2\pi}{T}\right)^2 A \quad ; \quad (g = \pi^2)$$

$$g = \frac{4\pi^2}{T^2} A \Rightarrow A = \frac{1}{4} = 0.25 \text{ m}$$

#### SECTION (B) : ENERGY ऊर्जा

- 2.# The potential energy of a particle of mass ' $m$ ' situated in a unidimensional potential field varies as  $U(x) = U_0 [1 - \cos ax]$ , where  $U_0$  and  $a$  are constants. The time period of small oscillations of the particle about the mean position :  
 $m$  द्रव्यमान का एक कण जो कि एकविमीय स्थितिज क्षेत्र में स्थित है, की स्थितिज ऊर्जा  $U(x) = U_0(1 - \cos ax)$  से परिवर्तित होती है, जहाँ  $U_0$  तथा  $a$  नियतांक हैं। माध्य अवस्था के सापेक्ष कण के लघु दोलों का आवर्त काल होगा—

(A)  $2\pi \sqrt{\frac{m}{a U_0}}$

(B)  $2\pi \sqrt{\frac{a m}{U_0}}$

(C\*)  $2\pi \sqrt{\frac{m}{a^2 U_0}}$

(D)  $2\pi \sqrt{\frac{a^2 m}{U_0}}$

Sol. Restoring force प्रत्यानयन बल  $F = \frac{-du}{dx} = \frac{-d}{dx} (u_0 (1 - \cos ax))$

$$F(x) = -u_0 a \sin ax$$

for small angle लघु कोण के लिये  $\sin ax \approx ax$

$$F = -u_0 a^2 x \Rightarrow \text{acc. त्वरण} = \frac{-u_0 a^2 x}{m} = -\omega^2 x = \left(\frac{2\pi}{T}\right)^2 x$$

$$\text{So, Time period आवर्तकाल } T = 2\pi \sqrt{\frac{m}{u_0 a^2}}$$

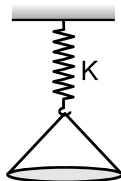


## SECTION (C) : SPRING MASS SYSTEM

## स्प्रिंग द्रव्यमान निकाय

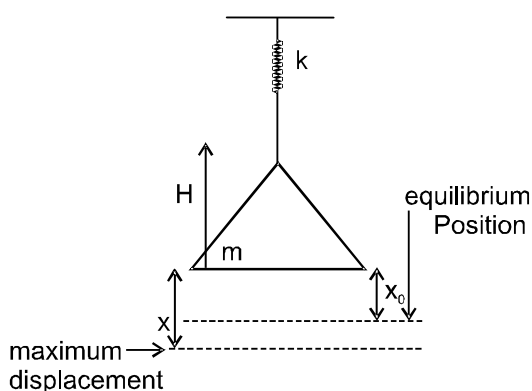
3. # A solid ball of mass  $m$  is made to fall from a height  $H$  on a pan suspended through a spring of spring constant  $K$  as shown in figure. If the ball does not rebound and the pan is massless, then amplitude of oscillation is

चित्रानुसार एक  $m$  द्रव्यमान की ठोस गेंद  $H$  ऊँचाई से एक चौड़े पलड़े पर गिरती है जो  $k$  स्प्रिंग नियतांक वाली स्प्रिंग से लटका है। यदि पलड़ा द्रव्यमान रहित तथा गेंद टकराकर वापस नहीं उछलती है, तो दोलन का आयाम होगा—



- (A)  $\frac{mg}{K}$  (B\*)  $\frac{mg}{k} \left(1 + \frac{2HK}{mg}\right)^{1/2}$  (C)  $\frac{mg}{K} + \left(\frac{2HK}{mg}\right)^{1/2}$  (D)  $\frac{mg}{K} \left[1 + \left(1 + \frac{2HK}{mg}\right)^{1/2}\right]$

Sol.



velocity before collision टकराने से पहले वेग =  $\sqrt{2gH}$

pan is massless so velocity after collision

बर्तन भी द्रव्यमानहीन है अतः टक्कर के बाद वेग

$$= \sqrt{2gH}$$

by energy conservation

ऊर्जा संरक्षण से

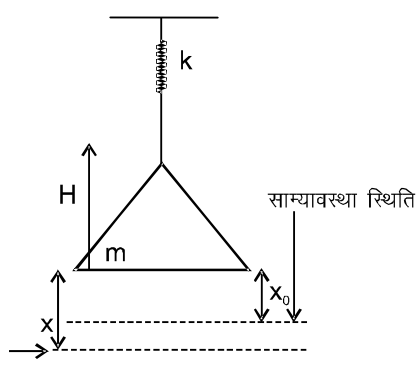
$$mg(x) + \frac{1}{2}m(\sqrt{2gH})^2 = \frac{1}{2}Kx^2$$

$$Kx^2 - 2mgx - 2mgH = 0$$

$$x = \frac{mg}{K} + \frac{mg}{K} \sqrt{1 + \frac{2HK}{mg}}$$

at equilibrium साम्यवस्था पर  $Kx_0 = mg \Rightarrow x_0 = mg/K$

$$\text{mean Amplitude माध्य आयाम} = x - x_0 = \frac{mg}{K} \sqrt{1 + \frac{2HK}{mg}}$$

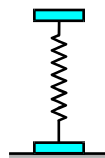




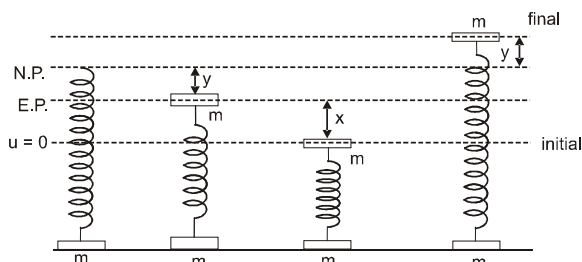


4. # Two plates of same mass are attached rigidly to the two ends of a spring as shown in figure. One of the plates rests on a horizontal surface and the other results a compression  $y$  of the spring when it is in equilibrium state. The further minimum compression required, so that after the force causing compression is removed the lower plate is lifted off the surface, will be:

चित्रानुसार दो समान द्रव्यमान की प्लेटें एक स्प्रिंग के दोनों सिरों से दृढ़ता पूर्वक जोड़ी जाती हैं। एक प्लेट क्षैतिज सतह पर विश्राम अवस्था में है जबकि दूसरी प्लेट के कारण स्प्रिंग में  $y$  संपीड़न है तथा यह स्थिर और साम्य अवस्था में है। वह न्यूनतम संपीड़न जिसके लिये संपीड़न बल को हटाने के पश्चात् नीचे वाली प्लेट गति के दौरान सतह से संपर्क छोड़ दे, होगा—

(A)  $0.5y$ (B)  $3y$ (C\*)  $2y$ (D)  $y$ 

Sol.



Let upper block is pushed down by  $x$ , at equilibrium  $mg = ky$ , i.e., weight of upper block is balanced by spring. When it is deformed by  $y$ , upper block will perform SHM with amplitude  $x$  about equilibrium position, lower block will leave surface when spring is extended by  $y$ , means upper block is at distance  $2y$  from its mean position. That should be upper extreme position of upper block. So amplitude  $x = 2y$  माना उच्च ब्लॉक नीचे  $x$  धकेला जाता है। साम्यवस्था पर  $mg = ky$  है अर्थात् उच्च ब्लॉक का भार स्प्रिंग द्वारा संतुलित होता है, जब स्प्रिंग में संकुचन  $y$  है। उच्च ब्लॉक साम्यवस्था के परितः आयाम  $x$  के साथ स.आ.ग. करेगा। जब स्प्रिंग  $y$  से विस्तारित होती है, निम्न ब्लॉक सतह को छोड़ देगा अर्थात् उच्च ब्लॉक इसकी माध्यवस्था से  $2y$  दूरी पर होता है। यह उच्च ब्लॉक की उच्च सीमान्त स्थिति होनी चाहिये अतः आयाम  $x = 2y$ .

**Alternative :**at equilibrium of upper block  $mg = ky$ साम्यवस्था पर उच्च ब्लॉक के लिये  $mg = ky$ lower plate will leave the surface if the extension in spring is  $y$ यदि स्प्रिंग में विस्तार  $y$  है तो निम्न प्लेट सतह छोड़ देगीLet upper plate is displaced by  $x$  downward and leftमाना उच्च प्लेट  $x$  नीचे विस्थापित करते हुये छोड़ी जाती है

so by energy conservation between compressed to extended positions

तो ऊर्जा संरक्षण से (संपीड़ित तथा विस्तारित अवस्थाओं के मध्य)

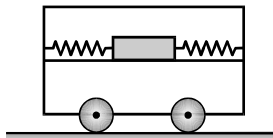
$$0 + \frac{1}{2} k (x + y)^2 = mg (x + 2y) + \frac{1}{2} ky^2$$

$$\Rightarrow \frac{1}{2} kx^2 + \frac{1}{2} ky^2 + kxy = mgx + mg2y + \frac{1}{2} ky^2 \Rightarrow x = 2y$$



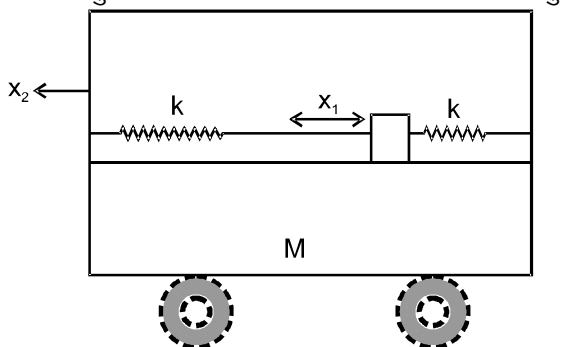
- 5.# Two springs, each of spring constant  $k$ , are attached to a block of mass  $m$  as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass  $M$ . If the block is displaced by  $x$  cm and released, the period of oscillation is :

दो समान स्प्रिंग जिनका स्प्रिंग नियतांक  $k$  है, चित्रानुसार  $m$  द्रव्यमान के गुटके से जुड़ी हुई है। गुटका एक क्षैतिज चिकने प्लेटफॉर्म पर खिसक सकता है। क्षैतिज प्लेटफॉर्म  $M$  द्रव्यमान की ट्राली की विपरीत दीवारों से जुड़ा है। यदि गुटके को  $x$  सेमी विस्थापित करके छोड़ दिया जाये तो दोलन का आवर्तकाल होगा—



- (A)  $T = 2\pi \sqrt{\frac{Mm}{2k}}$  (B)  $T = 2\pi \sqrt{\frac{(M+m)}{kmM}}$  (C\*)  $T = 2\pi \sqrt{\frac{mM}{2k(M+m)}}$  (D)  $T = 2\pi \frac{(M+m)^2}{k}$

Sol. Let displacement of block is  $x_1$  and of cart is  $x_2$  as shown  
माना वस्तु का विस्थापन  $x_1$  तथा गाड़ी का  $x_2$  है चित्रानुसार



by linear momentum conservation  
रेखीय संवेग संरक्षण से

$$mv_1 = Mv_2 \Rightarrow v_2 = \frac{mv_1}{M} \text{ so } x_2 = \frac{mx_1}{M}$$

For block Force equation can be written as

गुटके के लिये बल समीकरण ऐसे लिखी जा सकती है।

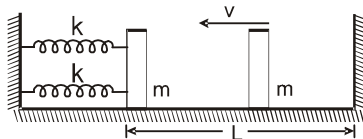
$$F = 2k(x_1 + x_2) = m\omega^2 x_1$$

$$\Rightarrow 2k \left( x_1 + \frac{m}{M} x_1 \right) = m\omega^2 x_1 \Rightarrow \omega^2 = 2k \left( \frac{M+m}{Mm} \right)$$

$$\text{So अतः } T = 2\pi \sqrt{\frac{Mm}{2k(M+m)}}$$

- 6.# The right block in figure moves at a speed  $V$  towards the left block placed in equilibrium. All the surfaces are smooth and all the collisions are elastic. Find the time period of periodic motion. Neglect the width of the blocks.

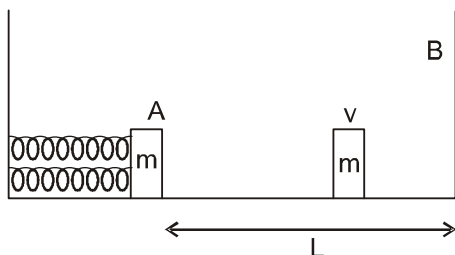
चित्रानुसार दांयी ओर स्थित गुटका  $v$  चाल से साम्यावस्था में स्थित बायें गुटके की ओर गति कर रहा है। सभी टक्करें प्रत्यास्थ हैं तथा सभी सतह घर्षण रहित है। आवर्त गति का आवर्तकाल ज्ञात करो ? दोनों गुटको की चौड़ाई नगण्य मानिये।



- (A\*)  $\pi \sqrt{\frac{m}{2k}} + \frac{2L}{v}$  (B)  $\pi \sqrt{\frac{m}{2k}} + \frac{L}{v}$  (C)  $\pi \sqrt{\frac{m}{2k}} - \frac{L}{v}$  (D)  $\pi \sqrt{\frac{m}{k}} + \frac{L}{v}$



Sol.



since all collision are elastic and total energy is conserved  
चूँकि सारी टक्करें प्रत्यास्थ हैं और कुल ऊर्जा संरक्षित है।

Time period of motion of right block is  $t_1 = \frac{2L}{v}$ .

दाँये गुटके का आवर्तकाल है  $t_1 = \frac{2L}{v}$ .

Time period of motion of left block is  $t_2 = \frac{1}{2} 2\pi \sqrt{\frac{m}{2k}}$

बाँये गुटके का आवर्तकाल है  $t_2 = \frac{1}{2} 2\pi \sqrt{\frac{m}{2k}}$

So, total time period अतः कुल आवर्त काल =  $\frac{2L}{v} + \frac{1}{2} \left[ 2\pi \sqrt{\frac{m}{2k}} \right]$

### SECTION (D) : SIMPLE PENDULUM सरल लोलक

7. The bob in a simple pendulum of length  $\ell$  is released at  $t = 0$  from the position of small angular displacement  $\theta_0$ . Linear displacement of the bob at any time  $t$  from the mean position is given by एक  $\ell$  लम्बाई के सरल लोलक के गोलक को  $t = 0$  समय पर अल्प कोणीय विस्थापन  $\theta_0$  से छोड़ा जाता है। किसी समय  $t$  पर माध्य स्थिति से गोलक का रेखीय विस्थापन होगा :

(A\*)  $\ell \theta_0 \cos \sqrt{\frac{g}{\ell}} t$  (B)  $\ell \sqrt{\frac{g}{\ell}} t \cos \theta_0$  (C)  $\ell g \sin \theta_0$  (D)  $\ell \theta_0 \sin \sqrt{\frac{g}{\ell}} t$

Sol. For simple pendulum  $\omega = \sqrt{g/\ell}$  and maximum linear displacement  $x_0 = \ell \theta_0$  and equation of S.H.M

सरल लोलक के लिये  $\omega = \sqrt{g/\ell}$  और अधिकतम रेखीय विस्थापन  $x_0 = \ell \theta_0$  और स.आ.ग. की समीकरण

$$x = x_0 \cos \omega t$$

$$x = \ell \theta_0 \cos \sqrt{\frac{g}{\ell}} t$$

8. The period of small oscillations of a simple pendulum of length  $\ell$  if its point of suspension O moves a with a constant acceleration  $\alpha = \alpha_1 \hat{i} - \alpha_2 \hat{j}$  with respect to earth is ( $\hat{i}$  and  $\hat{j}$  are unit vectors in horizontal and vertically upward directions respectively)  $\ell$  लम्बाई के सरल लोलक के अल्प दोलों का आवर्तकाल क्या होगा, यदि निलम्बन बिन्दु O पृथ्वी के सापेक्ष नियत त्वरण  $\alpha = \alpha_1 \hat{i} - \alpha_2 \hat{j}$  से गति करता है ( $\hat{i}$  तथा  $\hat{j}$  क्रमशः क्षैतिज तथा ऊर्ध्वाधर ऊपर की ओर दिशाओं में एकांक सदिश हैं।) –

(A\*)  $T = 2\pi \sqrt{\frac{\ell}{\{(g - \alpha_2)^2 + \alpha_1^2\}^{1/2}}}$

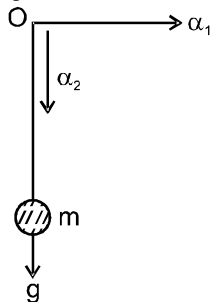
(B)  $T = 2\pi \sqrt{\frac{\ell}{\{(g - \alpha_1)^2 + \alpha_2^2\}^{1/2}}}$

(C)  $T = 2\pi \sqrt{\frac{\ell}{g}}$

(D)  $T = 2\pi \sqrt{\frac{\ell}{(g^2 + \alpha_1^2)^{1/2}}}$



**Sol.** Point O is moving as shown बिन्दु O चित्रानुसार गति कर रहा है।



So acc. Of particle w.r.t O

अतः कण का O के सापेक्ष त्वरण

$$= (-\alpha_1 \hat{i} + (\alpha_2 - g) \hat{j})$$

So अतः  $g_{\text{eff.}} = \sqrt{\alpha_1^2 + (g - \alpha_2)^2}$

So time period अतः आवर्तकाल

$$= 2\pi \sqrt{\frac{\ell}{(\alpha_1^2 + (g - \alpha_2)^2)^{1/2}}}$$

## SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

### पिण्ड लोलक व मरोड़ी लोलक

9. A simple pendulum ; a physical pendulum; a torsional pendulum and a spring–mass system, each of same frequency are taken to the Moon. If frequencies are measured on the moon, which system or systems will have it unchanged ?

चार प्रकार के दोलन निकाय, एक सरल लोलक, एक भौतिक लोलक, एक मरोड़ी लोलक तथा एक स्प्रिंग द्रव्यमान निकाय जिनका प्रत्येक की आवृत्ति समान है, को चन्द्रमा पर ले जाया जाता है। अगर चन्द्रमा पर आवृत्ति ज्ञात की जाए तो किन निकायों या निकाय में यह अपरिवर्तित होगा –

- (A\*) spring–mass system and torsional pendulum.  
(B) only spring–mass system.  
(C) spring–mass system and physical pendulum.  
(D) None of these

स्प्रिंग–द्रव्यमान निकाय तथा मरोड़ी लोलक में केवल स्प्रिंग द्रव्यमान निकाय में स्प्रिंग–द्रव्यमान निकाय तथा भौतिक दोलित्र में इनमें से कोई नहीं।

**Sol.** (B) For simple pendulum

$$T = 2\pi \sqrt{\frac{\ell}{g}} \left[ \begin{array}{l} \ell - \text{const} \\ g - \text{change} \end{array} \right]$$

For physical pendulum  $T = 2\pi \sqrt{\frac{I}{mg\ell}}$  (g–change)  $I, m, \ell \rightarrow \text{const}$

For Torsional pendulum  $T = 2\pi \sqrt{\frac{I}{C}} \left( \frac{I}{C} = \text{Const} \right)$  for any planet

For spring mass system  $T = 2\pi \sqrt{\frac{m}{K}}$

$$\therefore mg = k\ell \quad \left( \frac{m}{K} = \frac{\ell}{g} = \frac{\ell}{g'} = \text{Const for all planet} \right)$$

Both the spring–mass system & torsional pendulum have no dependence on gravitational acceleration for their frequencies .



**Sol. (B)** सरल लोलक के लिए  $T = 2\pi \sqrt{\frac{\ell}{g}}$   $\left[ \begin{array}{l} \ell - \text{नियत} \\ g - \text{परिवर्तित} \end{array} \right]$

भौतिक लोलक  $T = 2\pi \sqrt{\frac{I}{mg\ell}}$  ( $g$ -परिवर्तित)  $I, m, \ell \rightarrow \text{अचर}$

मरोड़ी लोलक  $T = 2\pi \sqrt{\frac{I}{C}}$   $\left( \frac{I}{C} = \text{Const} \right)$  सभी ग्रहों के लिए

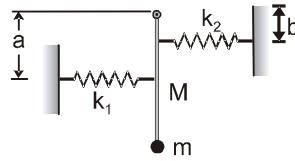
स्प्रिंग द्रव्यमान निकाय के लिए  $T = 2\pi \sqrt{\frac{m}{K}}$

$$\therefore mg = k\ell \quad \left( \frac{m}{K} = \frac{\ell}{g} = \frac{\ell}{g'} = \text{सभी ग्रहों के लिए अचर} \right)$$

स्प्रिंग द्रव्यमान निकाय तथा मरोड़ी लोलक दोनों की आवृत्ति गुरुत्वीय त्वरण पर निर्भर नहीं करता है।

- 10.#** A rod of mass  $M$  and length  $L$  is hinged at its one end and carries a particle of mass  $m$  at its lower end. A spring of force constant  $k_1$  is installed at distance  $a$  from the hinge and another of force constant  $k_2$  at a distance  $b$  as shown in the figure. If the whole arrangement rests on a smooth horizontal table top, the frequency of vibration is

द्रव्यमान  $M$  तथा लम्बाई  $L$  की एक छड़ एक सिरे से लटकी हुई है तथा दूसरे निचले सिरे पर एक  $m$  द्रव्यमान का कण जुड़ा हुआ है। एक  $K_1$  स्प्रिंग नियतांक की स्प्रिंग निलम्बित सिरे से  $a$  दूरी पर तथा  $K_2$  स्प्रिंग नियतांक की एक दूसरी स्प्रिंग निलम्बित सिरे से  $b$  दूरी पर चित्रानुसार लगायी गयी है। यदि सम्पूर्ण निकाय एक क्षैतिज चिकनी मेज पर रखा हो तो कम्पनों की आवृत्ति होगी—



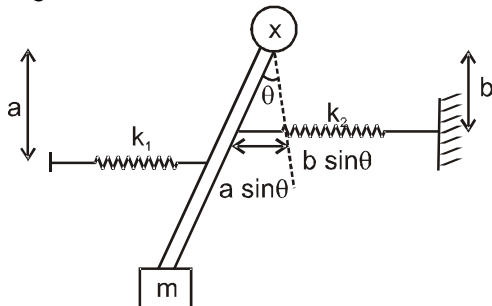
(A\*)  $\frac{1}{2\pi} \sqrt{\frac{k_1 a^2 + k_2 b^2}{L^2(m + \frac{M}{3})}}$

(B)  $\frac{1}{2\pi} \sqrt{\frac{k_2 + k_1}{M + m}}$

(C)  $\frac{1}{2\pi} \sqrt{\frac{k_2 + k_1 \frac{a^2}{b^2}}{\frac{4}{3} \frac{M}{3} + m}}$

(D)  $\frac{1}{2\pi} \sqrt{\frac{k_1 + \frac{k_2 b^2}{a^2}}{\frac{4}{3} \frac{m}{3} + M}}$

**Sol.** For small angular displacement ( $\theta$ )  
लघु कोणीय विस्थापन के लिये



Net torque on body वस्तु पर कुल बलाघूर्ण  $= I\alpha$

$$= (k_1 a \sin \theta) a + (k_2 b \sin \theta) b = \left( mL^2 + \frac{ML^2}{3} \right) \alpha$$

For small  $\theta$  लघु  $\theta$  के लिये  $\Rightarrow \alpha = \frac{k_1 a^2 + k_2 b^2}{mL^2 + \frac{ML^2}{3}} \theta$

$$\Rightarrow \text{frequency आवृत्ति} = \frac{1}{2\pi} \sqrt{\frac{k_1 a^2 + k_2 b^2}{L^2(m + M/3)}}$$



## SECTION (F) : SUPERPOSITION OF SHM सरल आवर्त गति का अध्यारोपण

11. A particle moves along the X-axis according to the equation  $x = 10 \sin^3(\pi t)$ . The amplitudes and frequencies of component SHMs are  
X-अक्ष पर एक कण समीकरण  $x = 10 \sin^3(\pi t)$  के अनुसार गति कर रहा है। सरल आवर्त गतियों के घटकों का आयाम तथा आवृत्तियाँ हैं –

(A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2 (B\*) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2  
(C) amplitude 10, 10 ; frequencies 1/2, 1/2 (D) amplitude 30/4, 10 ; frequencies 3/2, 2  
(A) आयाम 30/4, 10/4 ; आवृत्तियाँ 3/2, 1/2 (B\*) आयाम 30/4, 10/4 ; आवृत्तियाँ 1/2, 3/2  
(C) आयाम 10, 10 ; आवृत्तियाँ 1/2, 1/2 (D) आयाम 30/4, 10 ; आवृत्तियाँ 3/2, 2

**Sol.**  $x = 10 \sin^3(\pi t)$  ;  $\left( \sin^3 A = \frac{3 \sin A - \sin(3A)}{4} \right)$   
 $\Rightarrow x = 10 \left[ \frac{3 \sin(\pi t) - \sin(3\pi t)}{4} \right] \Rightarrow x = \frac{30}{4} \sin \pi t - \frac{10}{4} \sin 3\pi t$

So Amplitude अतः आयाम =  $\frac{30}{4}$  ,  $\frac{10}{4}$

frequency आवृत्ति = 1/2 , 3/2

12. The amplitude of a particle due to superposition of following S.H.Ms. Along the same line is

$X_1 = 2 \sin 50 \pi t$ ;  $X_2 = 10 \sin (50 \pi t + 37^\circ)$

$X_3 = -4 \sin 50 \pi t$ ;  $X_4 = -12 \cos 50 \pi t$

निम्नलिखित समान रेखा पर सरल आवर्त गतियों के अध्यारोपण द्वारा कण का आयाम होगा

$X_1 = 2 \sin 50 \pi t$ ;  $X_2 = 10 \sin (50 \pi t + 37^\circ)$

$X_3 = -4 \sin 50 \pi t$ ;  $X_4 = -12 \cos 50 \pi t$

(A)  $4\sqrt{2}$

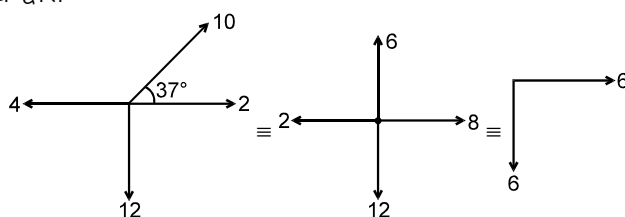
(B) 4

(C\*)  $6\sqrt{2}$

(D) none of these इनमें से कोई नहीं

**Hint :** Amplitude phasor diagram :

आयाम कला आरेख द्वारा



$\therefore$  resultant amplitude परिणामी आयाम =  $6\sqrt{2}$

13. When a body is suspended from a fixed point by a spring, the angular frequency of its vertical oscillations is  $\omega_1$ . When a different spring is used, the angular frequency is  $\omega_2$ . The angular frequency of vertical oscillations when both the springs are used together in series is given by

जब एक वस्तु को एक रस्सी द्वारा एक स्थिर बिन्दु से लटकाया जाता है तो इसके ऊर्ध्वाधर दोलों की कोणीय आवृत्ति  $\omega_1$  है। जब अलग रस्सी को काम में लेते हैं तो कोणीय आवृत्ति  $\omega_2$  है। जब दोनों रस्सियों को एक साथ श्रेणी में काम में लेते हैं, तो ऊर्ध्वाधर दोलों की कोणीय आवृत्ति दी जाती है :

(A)  $\omega = \left[ \omega_1^2 + \omega_2^2 \right]^{\frac{1}{2}}$  (B)  $\omega = \left[ \frac{\omega_1^2 + \omega_2^2}{2} \right]^{\frac{1}{2}}$  (C\*)  $\omega = \left[ \frac{\omega_1^2 \omega_2^2}{(\omega_1^2 + \omega_2^2)} \right]^{\frac{1}{2}}$  (D)  $\omega = \left[ \frac{\omega_1^2 \omega_2^2}{2(\omega_1^2 + \omega_2^2)} \right]^{\frac{1}{2}}$

**Ans.** (C)



**Sol.**  $\omega_1 = \sqrt{\frac{k_1}{m}}$   $\omega_2 = \sqrt{\frac{k_2}{m}}$

$$\omega = \sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}} = \sqrt{\frac{1}{m \left( \frac{1}{k_2} + \frac{1}{k_1} \right)}} = \sqrt{\frac{1}{\frac{1}{\omega_2^2} + \frac{1}{\omega_1^2}}} = \frac{\omega_1 \omega_2}{\sqrt{\omega_1^2 + \omega_2^2}}$$

- 14.** A particle performs simple harmonic motion at a frequency  $f$ . The frequency at which its kinetic energy varies is :

एक कण एक  $f$  आवृत्ति पर सरल आवर्त गति करता है। आवृत्ति जिस पर इसकी गतिज ऊर्जा बदलती है।

(A)  $f$

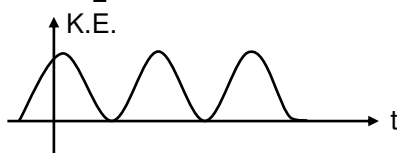
(B\*)  $2f$

(C)  $4f$

(D)  $\frac{f}{2}$

**Ans. (B)**

**Sol.**  $K.E. = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$



- 15.** A particle rests in equilibrium under two forces of repulsion whose centres are at distance of  $a$  and  $b$  from the particle. The forces vary as the cube of the distance. The forces per unit mass are  $k$  and  $k'$  respectively. If the particle be slightly displaced towards one of them the motion is simple harmonic with the time period equal to

एक कण प्रतिकर्षण के दो बलों के अन्दर साम्यावस्था में विराम करता है जिनके केन्द्र कण से  $a$  तथा  $b$  दूरी पर है। दोनों बल दूरी के घन के रूप में बदलते हैं। बलों के प्रति इकाई द्रव्यमान क्रमशः  $k$  तथा  $k'$  है। यदि कण को उनमें से किसी एक की तरफ थोड़ा सा विस्थापित कर दे तो गति सरल आवर्ती है, तो आवर्तकाल बराबर है –

(A)  $\frac{2\pi}{\sqrt{3 \left( \frac{k}{a^3} + \frac{k'}{b^3} \right)}}$

(B)  $\frac{2\pi}{\sqrt{\left( \frac{k}{a^3} + \frac{k'}{b^3} \right)}}$

(C)  $\frac{2\pi}{\sqrt{\left( \frac{k}{a^4} + \frac{k'}{b^4} \right)}}$

(D\*)  $\frac{2\pi}{\sqrt{3 \left( \frac{k}{a} + \frac{k'}{b} \right)}}$

**Ans. (D)**

**Sol.**  $C_1 a^3 = C_2 b^3$

$$\frac{C_1 a^3}{m} = k$$

$$\frac{C_2 b^3}{m} = k'$$

$$F_{rs} = C_1(a - x)^3 - C_2(b + x)^3$$

$$= C_1 a^3 \left( 1 - \frac{3x}{a} \right) - C_2 b^3 \left( 1 + \frac{3x}{b} \right)$$

$$= -3(C_1 a^2 + C_2 b^2)x$$

$$\therefore \text{time period} = \frac{2\pi}{\sqrt{3 \left( \frac{k}{a} + \frac{k'}{b} \right)}}$$





## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

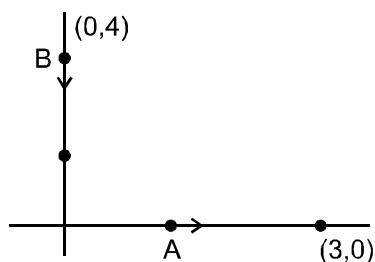
### भाग - II : एकल एवं द्वि-पूर्णांक मान प्रकार (SINGLE AND DOUBLE VALUE INTEGER TYPE)

#### SECTION (A) : EQUATION OF SHM सरल आवर्त गति का समीकरण

1. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the coordinates (3, 0) and (0, 4) respectively. At  $t = 0$ , B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. If the maximum and minimum distances between A and B is  $s_1$  and  $s_2$  then find  $s_1 + s_2$  (in cm).

दो कण A तथा B, x तथा y-अक्ष के अनुदिश समान आयाम तथा आवृत्ति क्रमशः 2 cm तथा 1 Hz से सरल आवर्त गति कर रहे हैं। A तथा B कणों की साम्यावस्था स्थितियाँ क्रमशः (3, 0) तथा (0, 4) निर्देशांकों पर हैं।  $t = 0$ , पर कण B साम्यावस्था पर है तथा मूल बिन्दु की तरफ जा रहा है और कण A मूलबिन्दु के सबसे निकट है तथा मूलबिन्दु से दूर जा रहा है। A तथा B के मध्य की अधिकतम तथा न्यूनतम दूरियाँ क्रमशः  $s_1$  व  $s_2$  हैं तब  $s_1 + s_2$  (cm में) ज्ञात करो।

Ans. 10  
Sol.



At  $t = 0$  Particle 2 is at point B and moving towards origin so displacement  
 $t = 0$  पर कण 2 बिन्दु B पर है तथा मूल बिन्दु की तरफ जा रहा है अतः

$$Y = 4 - A \sin \omega t$$

$$Y = 4 - 2 \sin \omega t$$

and displacement of particle 1 is

और कण 1 का विस्थापन है

$$X = 3 - A \cos \omega t$$

$$X = 3 - 2 \cos \omega t$$

So distance between them अतः उनके मध्य दूरी =  $\sqrt{X^2 + Y^2}$

$$s^2 = 29 - (16 \sin \omega t + 12 \cos \omega t) = 29 - 4 (4 \sin \omega t + 3 \cos \omega t)$$

$$\Rightarrow 29 - 20 (\sin \omega t + 37^\circ)$$

$$\text{So अतः } s_{\max}^2 = 49 \Rightarrow s_{\max} = 7 \text{ cm} = S_1$$

$$s_{\min}^2 = 9 \Rightarrow s_{\min} = 3 \text{ cm} = S_2 \quad \therefore S_1 + S_2 = 10 \text{ cm}$$

2. Two particles P and Q describe S.H.M. of same amplitude  $a$ , same frequency  $f$  along the same straight line from the same mean position. The maximum distance between the two particles is  $a\sqrt{2}$ . If the initial phase difference between the particles is  $\frac{\pi}{N}$  then find N:

दो कण P तथा Q समान आयाम  $a$  तथा समान आवृत्ति  $f$  के साथ समान माध्य स्थिति से सरल रेखा के अनुदिश सरल आवर्त गति करते हैं। कणों के मध्य अधिकतम दूरी  $a\sqrt{2}$  रहती है। कणों के मध्य प्रारम्भिक कलान्तर  $\frac{\pi}{N}$  है तब N ज्ञात कीजिए: —

Ans 2



**Sol.**  $x_1 = A \sin(\omega t + \phi_1)$   
 $x_2 = A \sin(\omega t + \phi_2)$

$$\Rightarrow |x_1 - x_2| = 2A \sin\left(2\omega t + \frac{\phi_1 + \phi_2}{2}\right) \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

To maximize अधिकतम के लिए  $|x_1 - x_2|$  :

$$\sin\left(2\omega t + \frac{\phi_1 + \phi_2}{2}\right) = 1$$

$$\Rightarrow a\sqrt{2} = 2a \times 1 \times \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \Rightarrow \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\Rightarrow \frac{\pi}{4} = \frac{\phi_1 - \phi_2}{2} \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{2} \therefore N = 2$$

3. A street car moves rectilinearly from station A (here car stops) to the next station B (here also car stops) with an acceleration varying according to the law  $f = a - bx$ , where  $a$  and  $b$  are positive constants and  $x$  is the distance from station A. If the maximum distance between the two stations is  $x = \frac{Na}{b}$  then find  $N$ .

स्थिरावस्था से एक कार स्टेशन A (यहाँ कार रुकती है) से अगले स्टेशन B (यहाँ भी कार रुकती है) तक  $f = a - bx$  के अनुसार परिवर्तित होने वाले त्वरण के साथ सरल रेखीय गति करती है। यहाँ  $a$  व  $b$  धनात्मक नियतांक हैं तथा  $x$ , स्टेशन A से दूरी है। दोनों स्टेशनों के बीच अधिकतम दूरी  $x = \frac{Na}{b}$  है तब  $N$  ज्ञात कीजिए –

**Ans.** 2

**Sol.**

$$f = a - bx$$

For maximum velocity, acceleration should be zero.

$$\text{i.e. } a - bx = 0 \Rightarrow x = \frac{a}{b}$$

$\therefore$  At  $x = \frac{a}{b}$ , the particle has its maximum velocity.

$$f = \frac{v dv}{dx} = a - bx \Rightarrow \frac{v^2}{2} = ax - \frac{bx^2}{2} + c$$

$$\text{At } x = 0 ; v = 0 \Rightarrow c = 0$$

Substituting ;  $x = \frac{a}{b}$  ; gives

$$v_{\max} = \frac{a}{\sqrt{b}}$$

Also, the velocity of the car should become zero at station B.

$$\text{i.e. } ax - \frac{bx^2}{2} = 0 \Rightarrow x = 0 ; x = \left(\frac{2a}{b}\right)$$

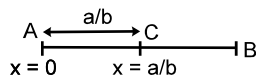
$\therefore$  Distance between the two stations is  $\frac{2a}{b}$

$\therefore N = 2$

**Alternate :**  $f = a - bx$  means particle will do SHM.

At mean position ;  $f = 0$

$$\Rightarrow x = \frac{a}{b}$$



In the figure shown, 'C' is the mean position and A & B are extreme positions

$$\therefore x_{\max} = \frac{2a}{b}$$

**Sol.**

$$f = a - bx$$

अधिकतम वेग के लिए त्वरण शून्य होना चाहिए

$$\text{i.e. } a - bx = 0 \Rightarrow x = \frac{a}{b}$$

 $\therefore x = \frac{a}{b}$  पर, कण अपनी अधिकतम चाल पर है।

$$f = \frac{v dv}{dx} = a - bx \Rightarrow \frac{v^2}{2} = ax - \frac{bx^2}{2} + c$$

$$x = 0 \text{ पर ; } v = 0 \Rightarrow c = 0$$

 प्रतिस्थापित करने पर ;  $x = \frac{a}{b}$  ; प्राप्त करते हैं।

$$v_{\max} = \frac{a}{\sqrt{b}}$$

तथा स्टेशन B पर कार का वेग शून्य हो जाएगा

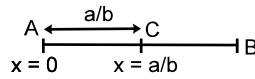
$$\text{i.e. } ax - \frac{bx^2}{2} = 0 \Rightarrow x = 0 ; x = \left( \frac{2a}{b} \right)$$

 $\therefore$  दोनों स्टेशनों के बीच की दूरी =  $\frac{2a}{b}$ 

$$\therefore N = 2$$

**विकल्प :**  $f = a - bx$  अर्थात् कण सरल आवर्त गति करेगा।माध्य स्थिति पर ;  $f = 0$ 

$$\Rightarrow x = \frac{a}{b}$$



प्रदर्शित चित्र में 'C' माध्य स्थिति है तथा A, B चरम स्थितियाँ हैं।

$$\therefore x_{\max} = \frac{2a}{b}$$

## SECTION (B) : ENERGY ऊर्जा

4. A particle is oscillating in a straight line about a centre O, with a force directed towards O. When at a distance 'x' from O, the force is  $mn^2x$  where 'm' is the mass and 'n' is a constant. The amplitude is  $a = 15 \text{ cm}$ . When at a distance  $\sqrt{3} \frac{a}{2}$  from O the particle receives a blow in the direction of motion which generates an extra velocity  $na$ . If the velocity is away from O at the time of blow and the new amplitude becomes  $k\sqrt{3} \text{ cm}$ , then find k.
- एक कण केन्द्र O के परितः एक सरल रेखा में दोलन करता है, जिस पर बल की दिशा O की ओर है। जब कण O से 'x' दूरी पर है तब कण पर बल  $mn^2x$  लगता है जहाँ 'm' द्रव्यमान व 'n' एक नियतांक है। आयाम  $a = 15 \text{ cm}$  है। जब कण O से दूरी  $\sqrt{3} \frac{a}{2}$  पर है तो कण गति की दिशा में एक झटका प्राप्त करता है जिससे अतिरिक्त वेग  $na$  प्राप्त होता है। यदि झटका प्राप्त करते समय वेग O से दूर की तरफ हो तो नया आयाम  $k\sqrt{3} \text{ cm}$  हो जाता है तो k ज्ञात कीजिए।

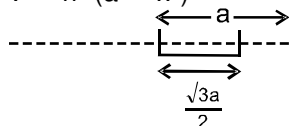
**Ans.** 15



**Sol.**  $\frac{1}{2} K x^2 + \frac{1}{2} m v^2 = \frac{1}{2} K A^2$

$$m n^2 x^2 + m v^2 = m n^2 a^2$$

$$v^2 = n^2 (a^2 - x^2)$$



$$\frac{1}{2} m (v + na)^2 + \frac{1}{2} K \left( \frac{\sqrt{3}a}{2} \right)^2 = \frac{1}{2} K A_1^2$$

$$\frac{1}{2} m v^2 + \frac{1}{2} m n^2 a^2 + \frac{2}{2} m n a v + \frac{1}{2} m n^2 \frac{3a^2}{4} = \frac{1}{2} m n^2 A_1^2$$

$$m n^2 (a^2 - x^2) + m n^2 a^2 + 2 m n a v + m n^2 \frac{3a^2}{4} = m n^2 A_1^2$$

$$a^2 - x^2 + a^2 + 2a \sqrt{a^2 - x^2} + \frac{3a^2}{4} = A_1^2$$

$$11a^2 - 4x^2 + 8a \sqrt{a^2 - 3a^2} = 4A_1^2$$

Putting  $x = \frac{\sqrt{3}}{2} a$  रखने पर

$$12a^2 = 4A_1^2 \quad A_1 = \sqrt{3}a = 15\sqrt{3} \text{ cm} \quad \therefore \quad k = 15$$

5. Two particles  $P_1$  and  $P_2$  are performing SHM along the same line about the same mean position. Initially they are at their positive extreme positions. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time after which the separation between the particles become 6 cm.

दो कण  $P_1$  तथा  $P_2$  एक ही रेखा के अनुदिश एक ही माध्य स्थिति के सापेक्ष सरल आवर्त गति कर रहे हैं। प्रारम्भ में यह धनात्मक चरम स्थितियों में होते हैं। अगर प्रत्येक कण का आवर्त काल 12 sec है तथा उनके आयाम का अन्तर 12 cm है तो वह न्यूनतम समय ज्ञात करो, जब कणों के बीच की दूरी 6 सेमी होती है –

**Sol.** The coordinates of the particles are

कण के निर्देशांक

$$x_1 = A_1 \cos \omega t, \quad x_2 = A_2 \cos \omega t$$

$$\text{separation बीच की दूरी} = x_1 - x_2 = (A_1 - A_2) \cos \omega t = 12 \cos \omega t$$

$$\text{Now अब } x_1 - x_2 = 6 = 12 \cos \omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{3}$$

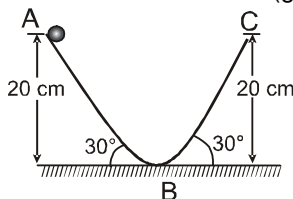
$$\Rightarrow \quad \frac{2\pi}{12} \cdot t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = 2s$$

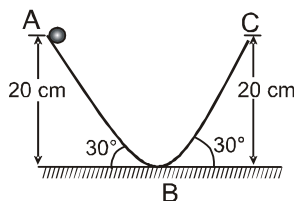
**Ans.  $t = 2s$**



- 6.# Assuming all the surfaces to be smooth, if the time period of motion of the ball is  $N \times 10^{-1}$  sec then find N. Neglect the small effect of bend near the bottom. ( $g = 10 \text{ m/s}^2$ )  
 सभी सतहों को चिकना मानते हुए यदि चित्र में दिखाये गये कण की गति का आवर्त काल  $N \times 10^{-1}$  sec है तो N ज्ञात करो। तली में मुड़े हुए छोटे भाग के प्रभाव को नगण्य मानना है। ( $g = 10 \text{ m/s}^2$ )



Ans. 16  
 Sol.



$$S = \frac{h}{\sin 30}, a = g \sin 30, h = 20 \text{ cm}, u = 0$$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2 \times (0.2 \times 2)}{10 \times (0.5)}} = 0.4$$

$$v = 0 + (g \sin 30)(0.4) = 2$$

For BC max height reached by the particle will be same as  $h = 20 \text{ cm}$   
 (by energy conservation)

and for BA and BC  $t = 0.4$

Total time period =  $0.4 \times 4 = 1.6 \text{ second} = 16 \times 10^{-1} \text{ sec}$

$\therefore N = 16$

BC के लिये कण द्वारा प्राप्त अधिकतम ऊँचाई  $h = 20 \text{ cm}$  के समान की है

(ऊर्जा संरक्षण से)

तथा BA एवं BC के लिये  $t = 0.4$

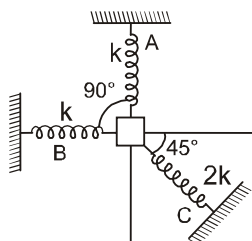
कुल आवर्तकाल =  $0.4 \times 4 = 1.6 \text{ second} = 16 \times 10^{-1} \text{ sec} \quad \therefore N = 16$

### SECTION (C) : SPRING MASS SYSTEM

### स्प्रिंग द्रव्यमान निकाय

- 7.# A block of mass  $m$  is attached to three springs A, B and C having force constants  $k$ ,  $k$  and  $2k$  respectively as shown in figure. If the block is slightly pushed against spring C. If the angular frequency of oscillations is  $\sqrt{\frac{Nk}{m}}$ , then find N. The system is placed on horizontal smooth surface.

चित्रानुसार बल नियतांक  $k$ ,  $k$  तथा  $2k$  वाली तीन स्प्रिंगों A, B व C से  $m$  द्रव्यमान का कण जुड़ा है। यदि कण को स्प्रिंग C के विरुद्ध दबाकर छोड़े तो दोलन की कोणीय आवृत्ति  $\sqrt{\frac{Nk}{m}}$  है तो N ज्ञात कीजिये। निकाय क्षैतिज चिकनी सतह पर स्थित है।



Ans. 3



**Resonance**  
 Educating for better tomorrow

Corporate Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in

Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 14



**Sol.** After displacement  $x$  against spring C  
स्प्रिंग C के विरुद्ध  $x$  विस्थापन के बाद

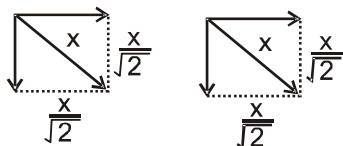


Figure-1

चित्र-1

Deformation in springs ABC is shown in figure 1.

चित्र - 1 में स्प्रिंगो A, B तथा C संकुचन दिखाया गया है।

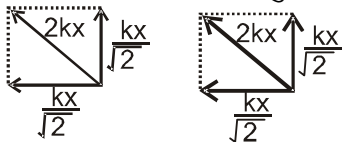


Figure-2

चित्र-2

Forces applied by three springs ABC is shown in figure-2.

चित्र-2 में तीनों स्प्रिंगो A, B, C के द्वारा आरोपित बल दर्शाया गया है।

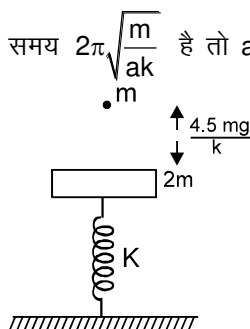
$$F = \frac{kx}{\sqrt{2}} \frac{1}{\sqrt{2}} \times 2 + 2kx = 3kx = m\omega^2 x \Rightarrow \omega = \sqrt{\frac{3k}{m}}$$

$$\therefore N = 3$$

**8.#**

In the figure shown mass  $2m$  is at rest and in equilibrium. A particle of mass  $m$  is released from height  $\frac{4.5mg}{k}$  from plate. The particle sticks to the plate. Neglecting the duration of collision. Starting from the time when the particle sticks to plate to the time when the spring is in maximum compression for the first time is  $2\pi\sqrt{\frac{m}{ak}}$  then find  $a$ .

चित्र में  $2m$  द्रव्यमान की प्लेट स्थिर तथा साम्यावस्था में है।  $m$  द्रव्यमान का एक कण प्लेट से  $\frac{4.5mg}{k}$  ऊँचाई से छोड़ा जाता है। कण प्लेट से चिपक जाता है। टक्कर के अन्तराल को नगण्य मानते हुए कण के प्लेट से चिपकने से लेकर स्प्रिंग में प्रथम बार अधिकतम सम्पीड़न तक का समय  $2\pi\sqrt{\frac{m}{ak}}$  है तो  $a$  ज्ञात करो।



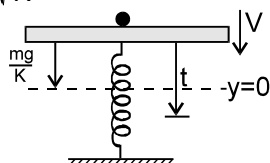
**Ans.** 3

**Sol.** Velocity of the particle just before collision

टक्कर के तुरन्त पहले कण का वेग

$$u = \sqrt{2g \times \frac{4.5mg}{K}}$$

$$u = 3g\sqrt{\frac{m}{K}}$$



Now it collides with the plate.



**Resonance**  
Educating for better tomorrow

Corporate Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in

Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 15



Now just after collision velocity of system of plate + particle

अब यह प्लेट से टकराता है

टक्कर के बाद (प्लेट + कण) निकाय का वेग

$$\mu = 3mv$$

$$\Rightarrow V = \frac{u}{3} = g\sqrt{\frac{m}{K}}$$

Now system perform's SHM with time period  $T = 2\pi\sqrt{\frac{3m}{K}}$  and mean position as  $\frac{mg}{K}$  distance below the point of collision.

Let the equation of motion be.

निकाय SHM करेगा इसका आवर्तकाल  $T = 2\pi\sqrt{\frac{3m}{K}}$  तथा माध्य स्थिति, टकराने वाली स्थिति से  $\frac{mg}{K}$  नीचे है।

गति की समीकरण

$$y = A \sin(\omega t + \phi)$$

for  $t = 0$  के लिए  $y = mg/K$

$$\frac{mg}{K} = A \sin \phi \quad \dots(1)$$

Now for amplitude अतः आयाम

$$V = \omega \sqrt{A^2 - x^2}$$

$$g\sqrt{\frac{m}{K}} = \sqrt{\frac{K}{3m}} \sqrt{A^2 - \frac{m^2 g^2}{K^2}}$$

$$\left(\sqrt{3} \frac{mg}{K}\right)^2 = A^2 - \frac{m^2 g^2}{K^2}$$

$$A = \frac{2mg}{K} \quad \dots(2)$$

By (1) & (2) समीकरण (1) व (2) से

$$T = 2\pi\sqrt{\frac{3m}{K}}$$

$$x = \frac{A}{2} \text{ to से } x = 0 \text{ तक} \Rightarrow t = \frac{T}{12}$$

$$x = 0 \text{ to से } x = A \text{ तक} \Rightarrow t = T/4$$

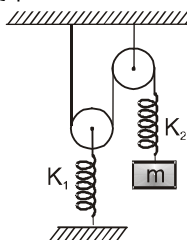
$$\text{total time कुल समय} = \frac{T}{12} + \frac{T}{4} = \frac{2\pi}{3} \sqrt{\frac{3m}{K}} = 2\pi\sqrt{\frac{m}{3K}}$$

$$\therefore a = 3$$

9. # For given spring mass system, If the time period of small oscillations of block about its mean position is

$\pi\sqrt{\frac{nm}{K}}$ , then find n. Assume ideal conditions. The system is in vertical plane and take  $K_1 = 2K$ ,  $K_2 = K$ .

दिये स्प्रिंग द्रव्यमान निकाय के लिए ब्लॉक का इसकी माध्य स्थिति के सापेक्ष अल्प दोलन का आवर्त काल  $\pi\sqrt{\frac{nm}{K}}$  है तो n ज्ञात करो ? सभी स्थितियां आदर्श है। निकाय ऊर्ध्वाधर तल में है तथा  $K_1 = 2K$ ,  $K_2 = K$  लेवें







**Ans.** 12

**Sol.** Let block is pushed down by  $x$  from its equilibrium position.  
माना ब्लॉक को इसकी साम्यवस्था स्थिति से  $x$  दूरी नीचे धकेला जाता है।

Let extension in springs be  $y_1$  and  $y_2$  as shown in figure.

माना चित्रानुसार स्प्रिंगों में विस्तार क्रमशः  $y_1$  तथा  $y_2$  है।

so restoring force on block  $F = K_2 y_2$

अतः ब्लॉक पर प्रत्यानयन बल  $F = K_2 y_2$

Force in left spring is बॉयी स्प्रिंग पर बल  $2F = K_1 y_1$

From constraint motion बंधित गति से  $2y_1 + y_2 = x$

$$\Rightarrow 2 \times \frac{2F}{K_1} + \frac{F}{K_2} = x$$

$$\Rightarrow F = \left( \frac{K_1 K_2}{K_1 + 4K_2} \right) x = \omega^2 x$$

$$\text{So अतः, } \omega = \sqrt{\frac{1}{m} \left( \frac{K_1 K_2}{K_1 + 4K_2} \right)} \Rightarrow T = 2\pi \sqrt{\frac{m (K_1 + 4K_2)}{K_1 K_2}}$$

$$T = 2\pi \sqrt{\frac{m (K_1 + 4K_2)}{K_1 K_2}} = \pi \sqrt{\frac{12m}{K}}$$

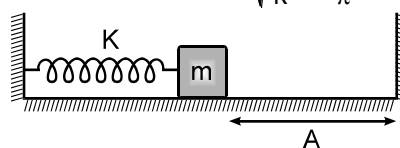
$$\therefore n = 12$$

**10. #** In the figure shown the spring is relaxed and mass  $m$  is attached to the spring. The spring is compressed by  $2A$  and released at  $t = 0$ . Mass  $m$  collides with the wall and loses two third of its kinetic energy and returns. Starting from  $t = 0$ , find the time taken by it to come back to rest again (instant at

which spring is again under maximum compression). Take  $\sqrt{\frac{m}{k}} = \frac{12}{\pi}$

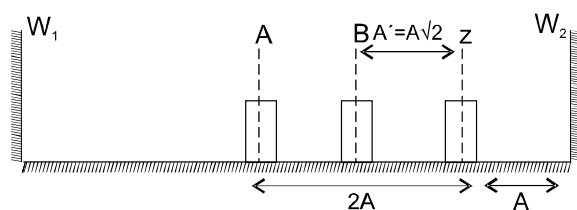
प्रदर्शित चित्र में स्प्रिंग सामान्य लम्बाई में है तथा द्रव्यमान  $m$  स्प्रिंग से जुड़ा है। स्प्रिंग को  $2A$  से सम्पीड़ित करके  $t = 0$  पर छोड़ दिया जाता है। द्रव्यमान  $m$  दीवार से टकराता है तथा अपनी गतिज ऊर्जा की दो तिहाई ऊर्जा ह्रास करके लोट जाता है।  $t = 0$  से प्रारम्भ करने के पश्चात् ब्लॉक द्वारा प्रथम बार विराम (वह क्षण जब दुबारा स्प्रिंग अधिकतम संपीड़न की

स्थिति में होती है) को प्राप्त करने में लगा समय ज्ञात करें। ( $\sqrt{\frac{m}{k}} = \frac{12}{\pi}$  लीजिये)



**Ans.** 17

**Sol.**



The motion starts from position A the time taken from A to  $W_2$  ( $t_1$ ) =  $\frac{T}{4} + \frac{T}{12}$

Before collision the energy of the system is conserved,

Kinetic energy of the block just before collision

$$K_i = \frac{1}{2} K (2A)^2 - \frac{1}{2} KA^2 = \frac{3}{2} KA^2 \text{ \& just after collision } K_f = \frac{K_i}{3} \text{ (given)} = \frac{1}{2} KA^2$$

Now during motion after collision, the energy is again conserved





**Sol.** Frequency (आवृत्ति)  $n_2 = \frac{1}{2\pi} \sqrt{\frac{k}{m_1 + m_2}}$

$$= \frac{1}{2\pi} \sqrt{\frac{100}{4+5}}$$

$$= \frac{1}{2\pi} \times \frac{10}{3} = \frac{5}{3\pi}$$

By conservation of linear momentum at mean position,  
माध्य अवस्था पर रेखीय संवेग संरक्षण से

$$P_i = P_f$$

$$\Rightarrow m_1 \omega_1 A_1 = (m_1 + m_2) \omega_2 A_2$$

$$\Rightarrow m_1 \sqrt{\frac{k}{m_1}} A_1 = (m_1 + m_2) \sqrt{\frac{k}{m_1 + m_2}} A_2$$

$$\Rightarrow \sqrt{k m_1} A_1 = \sqrt{k(m_1 + m_2)} A_2$$

$$\Rightarrow A_2 = \frac{2}{30} \text{ m.}$$

### SECTION (D) : SIMPLE PENDULUM सरल लोलक

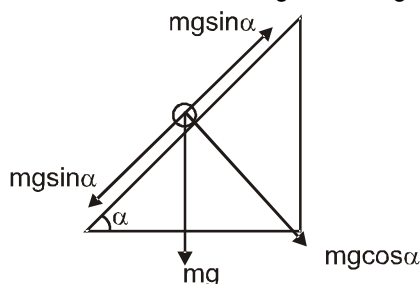
12. The period of oscillation of a simple pendulum of length  $L$  suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination  $\alpha = 60^\circ$  is given by  $\pi \sqrt{\frac{XL}{g}}$  then find  $X$ .

एक सरल लोलक जिसकी लम्बाई  $L$  है तथा यह  $\alpha = 60^\circ$  झुकाव वाले चिकने नततल पर नीचे की ओर गति करने वाले वाहन की छत पर लटका है, लोलक का आवर्त काल  $\pi \sqrt{\frac{XL}{g}}$  द्वारा दिया गया है तो  $X$  ज्ञात कीजिए –

**Ans** 8

**Sol.** Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows:

$\therefore$  Net tension in the string is  $T = mg \cos \alpha$



$$\text{So, } g_{\text{eff}} = \frac{T}{m} = \frac{mg \cos \alpha}{m} = g \cos \alpha$$

$$T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}} \quad \text{or} \quad T = 2\pi \sqrt{\frac{L}{g \cos \alpha}}$$

**Alternative :**

Whenever point of suspension is accelerating

$$\text{Take } T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}} \quad \text{Where } \vec{g}_{\text{eff}} = \vec{g} - \vec{a}$$

$\vec{a}$  = Acceleration of point of suspension.

In this question  $\vec{a} = g \sin \alpha$  (down the plane)

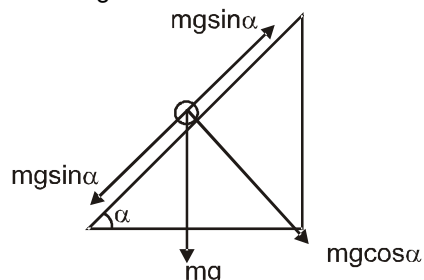


$$\therefore |\vec{g} - \vec{a}| = g_{\text{eff}} = \sqrt{g^2 + (g \sin \alpha)^2 + 2(g)(g \sin \alpha) \cos (90^\circ + \alpha)} = g \cos \alpha$$

$$\therefore T = 2\pi \sqrt{\frac{L}{g \cos \alpha}} \quad (\alpha = 60^\circ) \quad T = 2\pi \sqrt{\frac{2L}{g}} \quad \therefore x = 2$$

त्वरित निर्देश तंत्र के सापेक्ष पेन्डुलम के लोलक का मुक्त वस्तु रेखा चित्र ऐसा होगा।

$$\therefore \text{रस्सी में कुल तनाव } T = mg \cos \alpha$$



$$\text{और लोलक का कुल त्वरण } g_{\text{eff}} = \frac{T}{m} = \frac{mg \cos \alpha}{m} = g \cos \alpha$$

$$\therefore T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}} \quad \text{या} \quad T = 2\pi \sqrt{\frac{L}{g \cos \alpha}}$$

**वैकल्पिक**

जब भी आलम्बन बिन्दु त्वरित होता है।

$$\text{लीजिये } T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}} \quad \text{जहाँ } \vec{g}_{\text{eff}} = \vec{g} - \vec{a}$$

$\vec{a}$  = आलम्बन बिन्दु का त्वरण

इस प्रश्न में  $\vec{a} = g \sin \alpha$  (तल के नीचे की ओर)

$$\therefore |\vec{g} - \vec{a}| = g_{\text{eff}} = \sqrt{g^2 + (g \sin \alpha)^2 + 2(g)(g \sin \alpha) \cos (90^\circ + \alpha)} = g \cos \alpha$$

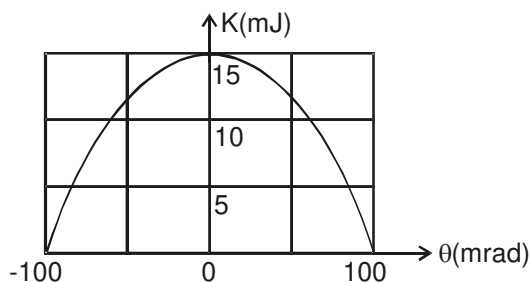
$$\therefore T = 2\pi \sqrt{\frac{L}{g \cos \alpha}} \quad (\alpha = 60^\circ)$$

$$T = 2\pi \sqrt{\frac{2L}{g}} \quad \therefore x = 2$$

**13. #** Figure shows the kinetic energy  $K$  of a simple pendulum versus its angle  $\theta$  from the vertical. The pendulum bob has mass 0.2 kg. If the length of the pendulum is equal to  $\frac{n}{g}$  meter, then find

$n$  ( $g = 10 \text{ m/s}^2$ ).

चित्र में सरल लोलक की गतिज ऊर्जा  $K$  तथा ऊर्ध्वाधर से कोण  $\theta$  के बीच ग्राफ दर्शाया गया है। लोलक का द्रव्यमान 0.2 किग्रा. है। यदि सरल लोलक की लम्बाई  $\frac{n}{g}$  मीटर के बराबर है तो  $n$  ज्ञात कीजिए - ( $g = 10 \text{ मी./से.}^2$ ).



**Ans. 15**



**Sol.**  $\frac{1}{2} m V_m^2 = 15 \times 10^{-3}$

$$V_m = \sqrt{0.150} \text{ m/s}$$

$$A\omega = \sqrt{0.150} \text{ m/s}$$

$$L\theta_m \cdot \sqrt{\frac{g}{L}} = \sqrt{0.150} \text{ m/s}$$

$$\sqrt{gL} = \frac{\sqrt{0.150}}{100 \times 10^{-3}} \quad gL = \frac{0.150}{0.01} = 1.5 \text{ m}$$

$$l = \frac{15}{g} \text{ m}$$

$$\therefore n = 15$$

- 14.** The bob of a simple pendulum executes simple harmonic motion in water with a period 't', while the period of oscillation of the bob is  $t_0$  in air. Neglecting frictional force of water and given that the density of the bob is  $(4/3) \times 1000 \text{ kg/m}^3$ . Find  $\frac{t}{t_0}$ .

किसी सरल लोलक का गोलक पानी में आवर्तकाल  $t$  के साथ सरल आवर्त गति करता है, जबकि इस गोलक के दोलन का वायु में आवर्तकाल  $t_0$  है। पानी का घर्षण बल नगण्य मानते हुए तथा गोलक का घनत्व  $(4/3) \times 1000 \text{ kg/m}^3$  दिया गया है।  $\frac{t}{t_0}$  ज्ञात कीजिए।

**Ans. 2**

**Sol.** The time period of simple pendulum in air  
वायु में सरल लोलक का आवर्त काल

$$T = t_0 = 2\pi \sqrt{\left(\frac{\ell}{g}\right)} \quad \dots\dots\dots (i)$$

$\ell$ , being the length of simple pendulum.

$\ell$ , सरल लोलक की लम्बाई है।

In water, effective weight of bob

जल में गोलक का प्रभावी भार

$w' = \text{weight of bob in air} - \text{upthrust}$

$w' = \text{वायु में गोलक का भार} - \text{ऊपर की ओर बल}$

$$\Rightarrow \rho V g_{\text{eff}} = mg - m'g$$

$$= \rho V g - \rho' V g = (\rho - \rho') V g$$

where  $\rho = \text{density of bob}$ , जहाँ  $\rho = \text{गोलक का घनत्व}$

$\rho' = \text{density of water}$  जल का घनत्व

$$\therefore g_{\text{eff}} = g \left( \frac{\rho - \rho'}{\rho} \right) = \left( 1 - \frac{\rho'}{\rho} \right) g$$

$$\therefore t = 2\pi \sqrt{\frac{\ell}{\left[ \left( 1 - \frac{\rho'}{\rho} \right) g \right]}} \quad \dots\dots\dots (ii)$$

$$\text{Thus अतः,} \quad \frac{t}{t_0} = \sqrt{\frac{1}{\left[ \left( 1 - \frac{\rho'}{\rho} \right) \right]}} = \sqrt{\frac{1}{1 - \frac{1000}{(4/3) \times 1000}}} = 2$$

$$\Rightarrow t = 2 t_0 ; \frac{t}{t_0} = 2 \quad \therefore N = 2$$



## SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

### पिण्ड लोलक व मरोड़ी लोलक

15. A solid sphere of radius  $R$  is half submerged in a liquid of density  $\rho$ . The sphere is slightly pushed down and released, If the frequency of small oscillations is  $\sqrt{\frac{3}{nR}}$ , then find  $n$ . Take  $\pi = \sqrt{g}$ .

$R$  त्रिज्या का एक ठोस गोला  $\rho$  घनत्व वाले द्रव में आधा डूबा है। यदि गोले को अल्प विस्थापित कर छोड़ा जाए और यह सरल आवर्त गति करने लगे तो गोले के दोलनों की आवृत्ति लघु विस्थापन के लिए  $\sqrt{\frac{3}{nR}}$  है तो  $n$  ज्ञात कीजिए।

$\pi = \sqrt{g}$  लीजिये।

**Ans. 8**

**Sol.** Half of the volume of sphere is submerged.

For equilibrium of sphere,  
weight = upthrust

$$\therefore V \rho_s g = \frac{V}{2} (\rho_L) (g) \quad \Rightarrow \quad \rho_s = \frac{\rho_L}{2}$$

When slightly pushed down by  $x$  weight will remain as it is while upthrust will increase. The increased upthrust will become the net restoring force (upwards).

$$F = -(\text{extra upthrust}) \\ = (\text{extra volume immersed}) (\rho_L) (g) \\ \text{or } ma = -(\pi R^2) x \rho_L g \quad (a = \text{acceleration})$$

$$\therefore \frac{4}{3} \pi R^3 \left( \frac{\rho_L}{2} \right) a = -(\pi R^2 \rho_L g) x \quad \therefore a = -\left( \frac{3g}{2R} \right) x$$

as  $a \propto -x$  motion is simple harmonic

Frequency of oscillation,

$$f = \frac{1}{2\pi} \sqrt{\frac{a}{x}} = \frac{1}{2\pi} \sqrt{\frac{3g}{2R}}$$

$$f = \sqrt{\frac{3}{8R}} \{ \pi = \sqrt{g} \} \quad \therefore N = 8$$

गोले का आधा आयतन डूबा हुआ है

गोले की साम्यावस्था के लिए

भार = उत्प्लावक बल

$$\therefore V \rho_s g = \frac{V}{2} (\rho_L) (g) \quad \Rightarrow \quad \rho_s = \frac{\rho_L}{2}$$

जब थोड़ा सा नीचे की ओर  $x$  से धकेला जाता है भार अपरिवर्तित रहता परन्तु उत्प्लावक बल बढ़ जाता है। बढ़ा हुआ उत्प्लावक बल परिणामी प्रत्यानयन बल (ऊपर की ओर) बन जायेगा।

$$F = -(\text{अतिरिक्त उत्प्लावक बल}) \\ = (\text{अतिरिक्त डूबा हुआ आयतन}) (\rho_L) (g)$$

$$\text{या } ma = -(\pi R^2) x \rho_L g \quad (a = \text{त्वरण})$$

$$\therefore \frac{4}{3} \pi R^3 a \left( \frac{\rho_L}{2} \right) = -(\pi R^2 \rho_L g) x \quad \therefore a = -\left( \frac{3g}{2R} \right) x$$

क्योंकि  $a \propto -x$  गति सरल आवर्त है

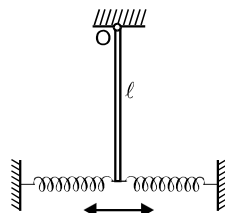
दोलन की आवृत्ति

$$f = \frac{1}{2\pi} \sqrt{\frac{a}{x}} = \frac{1}{2\pi} \sqrt{\frac{3g}{2R}}$$

$$f = \sqrt{\frac{3}{8R}} \{ \pi = \sqrt{g} \} \quad \therefore N = 8$$

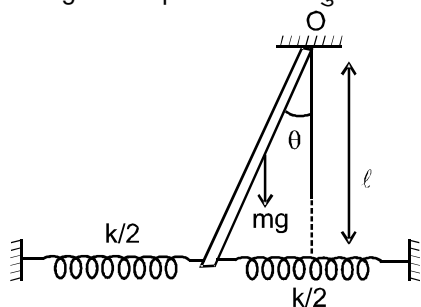


- 16.#** If the angular frequency of small oscillations of a thin uniform vertical rod of mass  $m$  and length  $\ell$  hinged at the point  $O$  (Fig.) is  $\sqrt{\frac{n}{\ell}}$ , then find  $n$ . The force constant for each spring is  $K/2$  and take  $K = \frac{2mg}{\ell}$ . The springs are of negligible mass. ( $g = 10 \text{ m/s}^2$ )  
 एक पतली एक समान ऊर्ध्वाधर छड़ जिसका द्रव्यमान  $m$  तथा लम्बाई  $\ell$  है, चित्रानुसार  $O$  बिन्दु पर कीलकीत है। इस छड़ के दोलन की आवृत्ति  $\sqrt{\frac{n}{\ell}}$  है तो  $n$  ज्ञात करो। प्रत्येक स्प्रिंग का बल नियतांक  $K/2$  है तथा  $K = \frac{2mg}{\ell}$  लीजिये। स्प्रिंगों के द्रव्यमान नगण्य है। ( $g = 10 \text{ m/s}^2$ )



**Ans.** 75

**Sol.** For small Angular displacement लघु कोणीय विस्थापन के लिये



Restoring torque about point  $O$  बिन्दु  $O$  के परितः प्रत्यानयन बलापूर्णा

$$(mg \frac{\ell}{2} \sin \theta) + (K\ell \sin \theta) \ell = \frac{m\ell^2}{3} \alpha, \quad (\text{for small } \theta \sin \theta \approx \theta) \quad (\text{लघु } \theta \text{ के लिये } \sin \theta \approx \theta)$$

$$\alpha = \frac{(mg \frac{\ell}{2} + K\ell^2)\theta}{\frac{m\ell^2}{3}} \Rightarrow \alpha = \left[ \frac{3g}{2\ell} + \frac{3k}{m} \right] \theta = -\omega^2 \theta$$

So angular speed अतः कोणीय वेग  $\omega = \sqrt{\left( \frac{3g}{2\ell} + \frac{3k}{m} \right)}$

$$\omega = \sqrt{\frac{3g}{2\ell} \left( 1 + \frac{2k\ell}{mg} \right)} = \sqrt{\frac{75}{\ell}} \quad \therefore N = 75$$

## PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

### भाग - III : एक या एक से अधिक सही विकल्प प्रकार

- 1.\*** A particle moves on the X-axis according to the equation  $x = x_0 \sin^2 \omega t$ . The motion is simple harmonic X-अक्ष पर एक कण समीकरण  $x = x_0 \sin^2 \omega t$  के अनुसार गति कर रहा है। सरल आवर्त गति का  
 (A\*) with amplitude  $x_0/2$  (B) with amplitude  $2x_0$  (C) with time period  $\frac{2\pi}{\omega}$  (D\*) with time period  $\frac{\pi}{\omega}$   
 (A\*) आयाम  $x_0/2$  है। (B) आयाम  $2x_0$  है। (C) आवर्त काल  $\frac{2\pi}{\omega}$  है। (D\*) आवर्त काल  $\frac{\pi}{\omega}$  है।



**Sol.**  $x = x_0 \sin^2 \omega t = \frac{x_0(1 - \cos 2\omega t)}{2}$

So अतः  $A = \frac{x_0}{2}$  Time period आवर्तकाल  $= \frac{2\pi}{\omega_1} = \frac{2\pi}{2\omega} = \pi/\omega$

**2.\*** Which of the following functions represent SHM?

निम्न में से कौनसे फलन सरल आवर्त गति प्रदर्शित करेंगे ?

(A\*)  $\sin 2\omega t$  (B\*)  $\sin^2 \omega t$  (C\*)  $\sin \omega t + 2 \cos \omega t$  (D)  $\sin \omega t + \cos 2\omega t$

**Sol.** Equation of S.H.M  $x - x_0 = a \sin (\omega t + \phi)$

$\Rightarrow x = x_0 + a \sin (\omega t + \phi)$

स.आ.ग. की समीकरण  $x - x_0 = a \sin (\omega t + \phi)$

$\Rightarrow x = x_0 + a \sin (\omega t + \phi)$

(A)  $x = \sin 2\omega t$

(B)  $x = \sin^2 \omega t = \left( \frac{1 - \cos 2\omega t}{2} \right) = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$

$\Rightarrow x - \frac{1}{2} = -\frac{1}{2} \cos 2\omega t$

(C)  $x = \sin \omega t + 2 \cos \omega t = \sin (\omega t + \phi) \quad \{\phi = \tan^{-1} 2\}$

So represent equation of S.H.M

अतः समीकरण स.आ.ग. की समीकरण को प्रस्तुत करती है

(D) But  $\sin \omega t + \cos 2\omega t$  cannot write as  $\sin (\omega t + \phi)$

$\sin \omega t + \cos 2\omega t$  को  $\sin (\omega t + \phi)$  के जैसे नहीं लिखा जा सकता है।

So it is not S.H.M equation

ये स.आ.ग. की समीकरण नहीं है।

**3.\*** The speed  $v$  of a particle moving along a straight line, when it is at a distance  $(x)$  from a fixed point of the line is given by  $v^2 = 108 - 9x^2$  (assuming mean position to have zero phase constant) (all quantities are in cgs units) :

एक कण जो कि एक सरल रेखा के अनुदिश गति कर रहा है जबकि यह कण सरल रेखा पर स्थित किसी निश्चित बिन्दु से  $x$  दूरी पर है तब कण की चाल  $v^2 = 108 - 9x^2$  से प्रदर्शित की जाती है (माध्य अवस्था पर शून्य कला नियतांक मानते हुए) (समस्त राशियाँ cgs पद्धति में हैं)

(A) the motion is uniformly accelerated along the straight line

गति, सरल रेखा के अनुदिश समरूप त्वरित होगी।

(B\*) the magnitude of the acceleration at a distance 3cm from the fixed point is  $27 \text{ cm/s}^2$

निश्चित बिन्दु से 3 सेमी की दूरी पर त्वरण का परिमाण  $27 \text{ सेमी/सेकण्ड}^2$  है।

(C\*) the motion is simple harmonic about the given fixed point.

दिये गये निश्चित बिन्दु के सापेक्ष गति सरल आवर्त गति है।

(D) the maximum displacement from the fixed point is 4 cm.

निश्चित बिन्दु से अधिकतम विस्थापन 4 सेमी है।

**Sol.**  $v^2 = 108 - 9x^2$

$\frac{2v dv}{dx} = -18x \Rightarrow \text{acc. } A = -9x \text{ (non-uniform) असमान}$

at  $x = 3 \text{ cm}$  पर

$a = -27$  or  $|a| = 27 \text{ cm/s}^2$

also  $a = -9x$  is a S.H.M equation so particle perform S.H.M about the give fixed point

तथा  $a = -9x$  एक स.आ.ग. समीकरण है अतः कण दिये गये स्थिर बिन्दु के सापेक्ष स.आ.ग. करेगा।

$V$  is maximum at  $x = 0$

$V$ ,  $x = 0$  पर अधिकतम है।

and  $V$  is Zero at  $x = \sqrt{12}$

तथा  $V$ ,  $x = \sqrt{12}$  पर शून्य है।

So Amplitude  $= 2\sqrt{3} \text{ cm}$

अतः आयाम  $= 2\sqrt{3} \text{ cm}$





- 4.\* A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :

एक क्षैतिज तख्ते पर एक आयताकार पिण्ड रखा है। तख्ता ऊर्ध्व दिशा में 40 सेमी. के आयाम से सरल आवर्ती रूप से दोलन करना प्रारम्भ करता है। यदि गति के दौरान तख्ते के क्षणिक रूप से स्थिरावस्था में आने पर पिण्ड का तख्ते से संबंध हट जाता है, तो :

(A\*) the period of oscillation is  $\left(\frac{2\pi}{5}\right)$  seconds    दोलनों का आवर्तकाल  $\left(\frac{2\pi}{5}\right)$  सेकण्ड है।

(B\*) the block weighs double its weight, when the plank is at one of the positions of momentary rest. तख्ते की किसी एक क्षणिक स्थिरावस्था की स्थिति में पिण्ड का वजन दुगुना होगा।

(C\*) the block weighs 0.5 times its weight on the plank halfway up पिण्ड का वजन आधा होगा जब तख्ता आधी ऊपरी ऊंचाई पर होगा।

(D\*) the block weighs 1.5 times its weight on the plank halfway down पिण्ड का वजन डेढ़ गुना होगा जब तख्ता नीचे की ओर आधी दूरी पर होगा।

(E\*) the block weights its true weight on the plank when the latter moves fastest पिण्ड का वजन इसका वास्तविक सत्य वजन होगा जब तख्ता तीव्रतम वेग से चल रहा होगा।

**Sol.** Given दिया है  $A = 0.4\text{m}$ , and और  $a = g$

$$\text{so अतः } \omega^2 A = g \Rightarrow \omega^2 = \frac{10}{0.4} = 25$$

$$\Rightarrow \omega = 5 \quad T = \frac{2\pi}{\omega} = 2\pi/5 \text{ sec. सैकण्ड}$$

At lowest position acceleration. निम्नतम बिन्दु पर त्वरण  $= \omega^2 A + g = g + g = 2g$

So weight अतः भार  $= m(2g) = 2mg$

at half distance आधी दूरी पर  $a = g/2$

So weight at upper half distance अतः भार ऊपरी आधी दूरी पर  $= m(g - g/2) = mg/2$

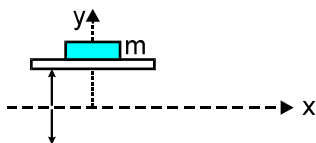
and weight at lower half distance तथा निचली आधी दूरी पर भार  $= m(g + g/2) = \frac{3mg}{2}$

actual weight at equilibrium position (maximum v)

सही भार साम्यावस्था स्थिति पर (अधिकतम वेग)

- 5\*.# As shown in figure a horizontal platform with a mass  $m$  placed on it is executing SHM along y-axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is : ( $g = 10 \text{ m/sec}^2 = \pi^2$ )

दिये गये चित्र में एक क्षैतिज आधार पर एक द्रव्यमान  $m$  रखा है तथा आधार Y अक्ष के अनुदिश सरल आवर्त गति करता है। यदि दोलनों का आयाम 2.5 सेमी है तो गति का न्यूनतम आवर्त काल क्या होगा ताकि द्रव्यमान आधार से अलग न हो, ( $g = 10 \text{ m/sec}^2 = \pi^2$  मानिये)



(A)  $\frac{10}{\pi} \text{ s}$

(B\*)  $\frac{\pi}{10} \text{ s}$

(C)  $\frac{\pi}{\sqrt{10}} \text{ s}$

(D\*)  $\frac{1}{\sqrt{10}} \text{ s.}$

**Sol.** acceleration त्वरण  $a = \omega^2 x$

$$\text{maximum acceleration अधिकतम त्वरण } a_{\max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 \frac{2.5}{100}$$

When block and platform are separated

जब वस्तु तथा प्लेटफार्म अलग होते हैं।

$$a_{\max} = g = 10$$

$$\frac{4\pi^2}{T^2} \cdot \frac{1}{40} = 10 \Rightarrow T^2 = \frac{\pi^2}{100} \quad T = \pi/10 \text{ sec}$$



- 6.\* For a body executing SHM with amplitude  $A$ , time period  $T$ , maximum velocity  $v_{\max}$  and phase constant zero, which of the following statements are correct for  $0 \leq t \leq \frac{T}{4}$  ( $y$  is displacement from mean position) ?

एक वस्तु, आयाम  $A$ , आवर्तकाल  $T$ , अधिकतम वेग  $v_{\max}$  तथा प्रारम्भिक कला स्थिरांक शून्य से सरल आवर्त गति करती है।

निम्न में से कौनसे कथन  $0 \leq t \leq \frac{T}{4}$  के लिए सत्य हैं (माध्य स्थिति से विस्थापन  $y$  है)–

(A\*) At  $y = (A/2)$ ,  $v > (v_{\max}/2)$

(B\*) for  $v = (v_{\max}/2)$ ,  $y > (A/2)$

(C\*) For  $t = (T/8)$ ,  $y > (A/2)$

(D\*) For  $y = (A/2)$ ,  $t < (T/8)$

(A\*)  $y = (A/2)$  पर  $v > (v_{\max}/2)$

(B\*)  $v = (v_{\max}/2)$  के लिए,  $y > (A/2)$

(C\*)  $t = (T/8)$  के लिए  $y > (A/2)$

(D\*)  $y = (A/2)$  के लिए  $t < (T/8)$

Sol. For S.H.M स.आ.ग. के लिये

$$y = A \sin \omega t = A \sin \left( \frac{2\pi t}{T} \right)$$

$$v = \omega \sqrt{A^2 - y^2} \quad v_{\max} = \omega A$$

(A) At  $y = A/2$  पर

$$v = \omega \sqrt{A^2 - A^2/4} = \frac{\sqrt{3}\omega A}{2} > \frac{\omega A}{2}$$

$$(B) \text{ For } v = \frac{v_{\max}}{2} \text{ के लिये } \Rightarrow \omega \sqrt{A^2 - y^2} = \frac{\omega A}{2}$$

$$\Rightarrow A^2 - y^2 = \frac{A^2}{4} \Rightarrow y = \frac{\sqrt{3}A}{2} > \frac{A}{2}$$

(C) for  $t = T/8$  के लिये

$$y = A \sin \left( \frac{2\pi \cdot T}{8 \cdot T} \right) = A/\sqrt{2} > \frac{A}{2}$$

$$(D) \text{ for } y = A/2 \text{ के लिये } A/2 = A \sin \left( \frac{2\pi t}{T} \right)$$

$$\Rightarrow \frac{2\pi t}{T} = \pi/6 \Rightarrow t = \frac{T}{12} < T/8$$

- 7.\* The potential energy of a particle of mass 0.1 kg, moving along the x-axis, is given by  $U = 5x(x - 4)$  J, where  $x$  is in meters. It can be concluded that

एक कण जिसका द्रव्यमान 0.1 किग्रा है,  $x$ -अक्ष के अनुदिश गति करता है। इसकी स्थितिज ऊर्जा  $U = 5x(x - 4)$  J, समीकरण द्वारा दी जाती है, जहां  $x$  मीटर में है। यह निष्कर्ष निकाला जा सकता है कि –

(A) the particle is acted upon by a constant force

कण पर नियत बल लग रहा है।

(B\*) the speed of the particle is maximum at  $x = 2$  m

कण की चाल  $x = 2$  मी. पर अधिकतम है।

(C\*) the particle executes SHM

कण सरल आवर्त गति कर रहा है।

(D\*) the period of oscillation of the particle is  $(\pi/5)$  sec

कण के दोलों का आवर्तकाल  $\pi/5$  सैकण्ड है।



**Sol.**  $F = \frac{-du}{dx} = -[10x - 20] = 20 - 10x$

acceleration त्वरण  $a = \frac{20 - 10x}{m} = 100(2 - x) = -\omega^2(x - 2)$

$a = 0$  at  $x = 2$  So  $V$  is maximum at  $x = 2$

$x = 2$  पर  $a = 0$  है अतः  $V$ ,  $x = 2$  पर अधिकतम है।

This is equation of S.H.M so particle executes S.H.M

यह स.आ.ग. की समीकरण है अतः कण स.आ.ग. करता है।

also तथा  $\omega^2 = 100 \Rightarrow \omega = 10$

$$T = \frac{2\pi}{\omega} = \pi/5 \text{ sec.}$$

- 8.\*** A particle free to move along the x-axis has potential energy given by  $U(x) = k[1 - e^{-x^2}]$  for  $-\infty \leq x \leq +\infty$ , where  $k$  is a positive constant of appropriate dimensions. Then select the incorrect options:

एक कण जो कि x-अक्ष के अनुदिश गति करने के लिए स्वतंत्र है कि स्थितिज ऊर्जा  $-\infty \leq x \leq +\infty$  के लिए समीकरण

$U(x) = k[1 - e^{-x^2}]$  से प्रदर्शित है, जहाँ  $k$  उपयुक्त विमाओं के साथ घनात्मक नियतांक है तो असत्य कथनों का चुनिये :

(A\*) at points away from the origin, the particle is in unstable equilibrium.

मूल बिन्दु से दूर वाले बिन्दुओं पर कण अस्थायी साम्यावस्था में होगा।

(B\*) for any finite non-zero value of  $x$ , there is a force directed away from the origin.

$x$  के किसी भी निश्चित अशून्य मान के लिए हमेशा एक बल मूल बिन्दु से दूर लगेगा।

(C\*) if its total mechanical energy is  $k/2$ , it has its minimum kinetic energy at the origin.

यदि इसकी कुल यांत्रिक ऊर्जा  $k/2$  है तो इसकी निम्नतम गतिज ऊर्जा मूल बिन्दु पर होगी।

(D) for small displacements from  $x = 0$ , the motion is simple harmonic.

$x = 0$  से लघु विस्थापनों के लिए, गति सरल आवर्त गति है।

**Sol.**  $U(x) = 1 - e^{-x^2}$

It is an exponentially increasing graph of potential energy ( $U$ ) with  $x^2$ . Therefore  $U$  versus  $x$  graph will be as shown.

From the graph it is clear that at origin

Potential energy  $U$  is minimum ( therefore, kinetic energy will be maximum ) and force acting on the

particle is also zero because  $F = \frac{-dU}{dx} = -(\text{slope of } U - x \text{ graph}) = 0$ .

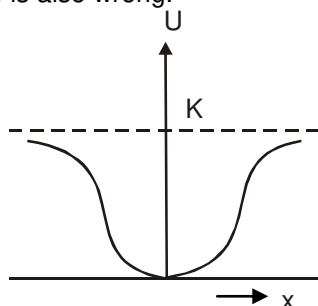
Therefore, origin is the stable equilibrium position. Hence particle will oscillate simple harmonically about  $x = 0$  for small displacements. Therefore, correct option is (D).

(A) At equilibrium position  $F = \frac{-dU}{dx} = 0$  i.e. slope of  $U$ - $X$  graph should be zero and from the graph we

can see that slope is zero at  $x = 0$  and  $x = \pm \infty$ . Now among these equilibriums stable equilibrium position is that where  $U$  is minimum ( Here  $x = 0$  ). Unstable equilibrium position is that where  $U$  is maximum ( Here none ). Neutral equilibrium position is that where  $U$  is constant ( Here  $x = \pm \infty$  ). Therefore, option (A) is wrong.

(B) For any finite non-zero value of  $x$ , force is directed towards the origin, because origin is in stable equilibrium position. Therefore, option (B) is incorrect.

(C) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (C) is also wrong.





**SOL.**  $U(x) = K(1 - e^{-x^2})$

यह स्थितिज ऊर्जा (U) का  $x^2$  के साथ चरघातांकी बढ़ता हुआ ग्राफ है। अतः U और X के मध्य ग्राफ चित्रानुसार होगा। ग्राफ से यह स्पष्ट है कि मूलबिन्दु पर स्थितिज ऊर्जा U न्यूनतम है (अतः गतिज ऊर्जा अधिकतम होगी) और कण पर

लगने वाला बल भी शून्य होगा क्योंकि  $F = \frac{-dU}{dx} = -(U - x \text{ ग्राफ का ढाल}) = 0$

अतः मूलबिन्दु स्थायी साम्यावस्था की स्थिति है अतः कण  $x = 0$  के सापेक्ष छोटे विस्थापन के लिये कण सरल आवर्त गति करेगा। अतः विकल्प (D) सही है।

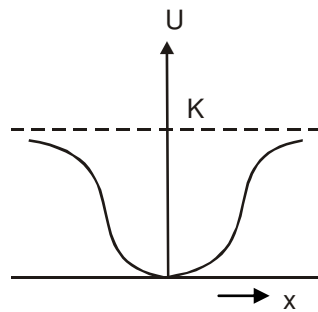
(A) साम्यवस्था पर  $F = \frac{-dU}{dx} = 0$  है अतः U-X ग्राफ का ढाल शून्य होगा और ग्राफ से हम कह सकते हैं कि  $x = 0$  तथा

$x = \pm \infty$  पर ढाल शून्य है। अब इन साम्यावस्थाओं में से स्थायी साम्यावस्था पर U का मान न्यूनतम होगा। (यहाँ  $x = 0$ ) अस्थायी साम्यावस्था पर U अधिकतम होती है। (यहाँ पर कोई नहीं) तथा उदासीन साम्यवास्था पर U नियत (यहाँ  $x = \pm \infty$ ) होती है अतः विकल्प (A) गलत है।

(B)  $x$  के किसी भी परिमित अशून्य मान के लिए बल मूलबिन्दु की ओर होगा क्योंकि मूलबिन्दु स्थायी साम्यावस्था की स्थिति है। अतः विकल्प (B) गलत है।

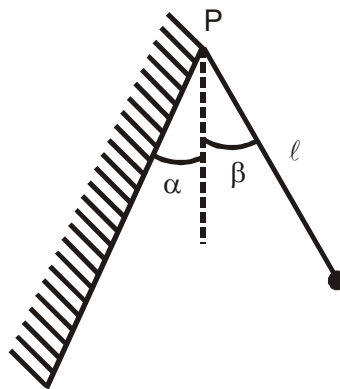
(C) मूलबिन्दु पर स्थितिज ऊर्जा न्यूनतम है अतः गतिज ऊर्जा अधिकतम होगी।

अतः विकल्प (C) भी गलत है।



**9.\*#** A ball is hung vertically by a thread of length  $\ell$  from a point 'P' of an inclined wall that makes an angle ' $\alpha$ ' with the vertical. The thread with the ball is then deviated through a small angle ' $\beta$ ' ( $\beta > \alpha$ ) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are

एक झुकी हुई दीवार जो कि ऊर्ध्व से ' $\alpha$ ' कोण बनाती है, के एक बिन्दु P से एक  $\ell$  लम्बाई की रस्सी द्वारा एक गेंद को लटकाया जाता है। रस्सी को गेंद सहित अल्प कोण ' $\beta$ ' ( $\beta > \alpha$ ) द्वारा विस्थापित करके स्वतंत्र छोड़ते हैं। दीवार को पूर्णतः प्रत्यास्थ मानते हैं। इस प्रकार के लोलक का आवर्तकाल होगा/होगे—



(A)  $2\sqrt{\frac{\ell}{g}} \left[ \sin^{-1} \left( \frac{\alpha}{\beta} \right) \right]$

(B\*)  $2\sqrt{\frac{\ell}{g}} \left[ \frac{\pi}{2} + \sin^{-1} \left( \frac{\alpha}{\beta} \right) \right]$

(C)  $2\sqrt{\frac{\ell}{g}} \left[ \cos^{-1} \left( \frac{\alpha}{\beta} \right) \right]$

(D\*)  $2\sqrt{\frac{\ell}{g}} \left[ \cos^{-1} \left( -\frac{\alpha}{\beta} \right) \right]$





**Sol.**  $\max^m$  amplitude अधिकतम आयाम  $= \beta \ell$

So time to travel angle  $\alpha$  अतः  $\alpha$  कोण तय करने में लगा समय

$$x = x_0 \sin \omega t$$

$$\alpha \ell = \beta \ell \sin \omega t$$

$$t = \frac{1}{\omega} \sin^{-1} \frac{\alpha}{\beta}$$

Now period of pendulum  $= \frac{1}{2}$  (actual time period) + 2 (time to travel angle  $\alpha$ )

अब लोलक का आवर्तकाल  $= \frac{1}{2}$  (वास्तविक आवर्तकाल) + 2 ( $\alpha$  कोण तय करने में लगा समय)

$$= \frac{1}{2} \cdot 2\pi \sqrt{\frac{\ell}{g}} + 2 \sqrt{\frac{\ell}{g}} \sin^{-1} \frac{\alpha}{\beta}$$

$$= 2 \sqrt{\frac{\ell}{g}} \left[ \pi/2 + \sin^{-1} \left( \frac{\alpha}{\beta} \right) \right]$$

Also if time taken to travel from right extreme position to wall is  $t$  then

यदि दायाँ सीमान्त स्थिति से दीवार तक जाने में लगा समय  $t$  है तो

$$-\alpha = \beta \cos \omega t$$

$$\Rightarrow \cos \omega t = -\frac{\alpha}{\beta}$$

$$t = \frac{1}{\omega} \cos^{-1} \left( \frac{-\alpha}{\beta} \right)$$

$$T = 2t = \frac{2}{\omega} \cos^{-1} \left( \frac{-\alpha}{\beta} \right)$$

$$T = 2 \sqrt{\frac{\ell}{g}} \cos^{-1} \left( \frac{-\alpha}{\beta} \right)$$

**10.\*** If a SHM is given by  $y = (\sin \omega t + \cos \omega t)$  m, which of the following statements are true?

यदि एक सरल आवर्त गति को  $y = (\sin \omega t + \cos \omega t)$  m द्वारा प्रदर्शित करें तो निम्न से कौनसे कथन सत्य है।

(A) The amplitude is 1 m आयाम का मान 1 मी. है।

(B\*) The amplitude is  $\sqrt{2}$  m आयाम का मान  $\sqrt{2}$  मी. है।

(C\*) Time is considered from  $y = 1$  m  $y = 1$  मी. से समय प्रारम्भ होगा।

(D) Time is considered from  $y = 0$  m  $y = 0$  मी. से समय प्रारम्भ होगा।

**Sol.** Given दिया है  $y = (\sin \omega t + \cos \omega t)$   $y = \sqrt{2} \sin(\omega t + \pi/4)$

So अतः  $A = \sqrt{2}$  m and तथा at  $t = 0$  पर  $y = 1$  m

**11.\*** The position of a particle at time  $t$  moving in x-y plane is given by  $\vec{r} = (\hat{i} + 2\hat{j}) A \cos \omega t$ . Then, the motion of the particle is :

x - y तल में गति करते हुए एक कण का स्थिति सदिश समय के साथ समीकरण  $\vec{r} = (\hat{i} + 2\hat{j}) A \cos \omega t$  के अनुसार बदलता है। कण की गति होगी -

(A\*) on a straight line एक सरल रेखा पर

(B) on an ellipse एक दीर्घ वृत्त पर

(C\*) periodic आवर्ती

(D\*) SHM सरल आवर्ती



**Sol.**  $\vec{r} = A \cos \omega t \hat{i} + 2A \cos \omega t \hat{j} \Rightarrow x = A \cos \omega t, y = 2A \cos \omega t$  so अतः  $y = 2x$

$$|\vec{r}| = A \cos \omega t \cdot \sqrt{5} = \sqrt{5} A \cos \omega t$$

So motion is at straight line, periodic and S.H.M

अतः गति सरल रेखा पर आवर्ती व स.आ.ग. है।

**12\*.** Three simple harmonic motions in the same direction having the same amplitude  $a$  and same period are superposed. If each differs in phase from the next by  $45^\circ$ , then, तीन सरल आवर्त गतियां जो कि समान दिशा में हैं तथा इनका आयाम  $a$  व आवर्तकाल समान है, एक दूसरे पर अध्यारोपित होती हैं। यदि प्रत्येक अपने अगले वाली से  $45^\circ$  कलान्तर से भिन्न है, तो :

(A\*) the resultant amplitude is  $(1+\sqrt{2})a$

परिणामी आयाम  $(1+\sqrt{2})a$  होगा।

(B) the phase of the resultant motion relative to the first is  $90^\circ$ .

परिणामी गति की कला प्रथम के सापेक्ष  $90^\circ$  पर होगी।

(C\*) the energy associated with the resulting motion is  $(3+2\sqrt{2})$  times the energy associated with any single motion.

परिणामी गति से सम्बन्धित ऊर्जा किसी भी एक गति से सम्बन्धित ऊर्जा की  $(3+2\sqrt{2})$  गुना होगी।

(D) the resulting motion is not simple harmonic.

परिणामी गति सरल आवर्ती नहीं होगी।

**Sol.** From superposition principle

$$\begin{aligned} \vec{y} &= \vec{y}_1 + \vec{y}_2 + \vec{y}_3 \\ &= a \sin \omega t + a \sin (\omega t + 45^\circ) + a \sin (\omega t + 90^\circ) \\ &= a \{ \sin \omega t + \sin (\omega t + 90^\circ) \} + a \sin (\omega t + 45^\circ) \\ &= 2a \sin (\omega t + 45^\circ) \cos 45^\circ + a \sin (\omega t + 45^\circ) \\ &= (\sqrt{2} + 1) a \sin (\omega t + 45^\circ) \\ &= A \sin (\omega t + 45^\circ) \end{aligned}$$

Therefore, resultant motion is simple harmonic of amplitude

$$A = (\sqrt{2} + 1) a$$

and which differ in phase by  $45^\circ$  relative to the first.

$$\text{Energy in SHM (amplitude)}^2 \quad [E = \frac{1}{2} m A^2 \omega^2]$$

$$\therefore \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left( \frac{A}{a} \right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2})$$

$$\therefore E_{\text{resultant}} = (3 + 2\sqrt{2}) E_{\text{single}}$$

$$\text{अध्यारोपण सिद्धान्त से } \vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3$$

$$\begin{aligned} &= a \sin \omega t + a \sin (\omega t + 45^\circ) + a \sin (\omega t + 90^\circ) \\ &= a \{ \sin \omega t + \sin (\omega t + 90^\circ) \} + a \sin (\omega t + 45^\circ) \\ &= 2a \sin (\omega t + 45^\circ) \cos 45^\circ + a \sin (\omega t + 45^\circ) \\ &= (\sqrt{2} + 1) a \sin (\omega t + 45^\circ) \\ &= A \sin (\omega t + 45^\circ) \end{aligned}$$

अतः परिणामी गति सरल आवर्त गति है जिसका आयाम

$$A = (\sqrt{2} + 1) a$$

और यह प्रथम के सापेक्ष  $45^\circ$  कलान्तर पर है।

$$\text{SHM में ऊर्जा (आयाम)}^2 \quad [E = \frac{1}{2} m A^2 \omega^2]$$

$$\therefore \frac{E_{\text{परिणामी}}}{E_{\text{एकल}}} = \left( \frac{A}{a} \right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2}) \quad \therefore E_{\text{परिणामी}} = (3 + 2\sqrt{2}) E_{\text{एकल}}$$



## PART - IV : COMPREHENSION

### भाग - IV : अनुच्छेद (COMPREHENSION)

#### Comprehension # 1

A 2kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of  $5 \text{ m/s}^2$  when the acceleration suddenly ceases at time  $t = 0$  and the car moves upward with constant speed. ( $g = 10 \text{ m/s}^2$ )

बल नियतांक 400 N/m की स्प्रिंग के निचले सिरे से 2 किग्रा. का पिण्ड बिना कम्पन्न के लटका है। स्प्रिंग का शीर्ष सिरा लिफ्ट की छत से जुड़ा है। लिफ्ट  $5 \text{ मी./से.}^2$  के ऊपरी त्वरण से उठ रही है। जब  $t = 0$  पर त्वरण अचानक खत्म होता है, लिफ्ट एक समान चाल से ऊपर की ओर गतिमान है। ( $g = 10 \text{ मी./से.}^2$ )

1. What is the angular frequency of oscillation of the block after the acceleration ceases?

त्वरण समाप्त होने के पश्चात् पिण्ड की कोणीय आवृत्ति क्या है –

- (A\*)  $10\sqrt{2} \text{ rad/s}$  (B)  $20 \text{ rad/s}$  (C)  $20\sqrt{2} \text{ rad/s}$  (D)  $32 \text{ rad/s}$

**Sol.**  $\omega = \sqrt{\frac{K}{m}} = \sqrt{200} \text{ rad/s}$

2. The amplitude of the oscillations is

कम्पन्न का आयाम है –

- (A) 7.5 cm (B) 5 cm (C\*) 2.5 cm (D) 1 cm

3. The initial phase angle observed by an observer in the elevator, taking upward direction to be positive and positive extreme position to have  $\pi/2$  phase, is equal to

लिफ्ट में सवार व्यक्ति द्वारा प्रेक्षित प्रारम्भिक कला कोण है, जब कि ऊपरी दिशा को धनात्मक लें तथा धनात्मक सीमान्त अवस्था को  $\pi/2$  कला कोण स्थिरांक पर मानें –

- (A)  $-\pi/4 \text{ rad}$  (B)  $\pi/2 \text{ rad}$  (C)  $\pi \text{ rad}$  (D\*)  $3\pi/2 \text{ rad}$

#### Sol. 1 to 3

Maximum extension the spring from natural position is  $x$ .

Then  $mg + ma = kx$

$$\Rightarrow x = \frac{2(10+5)}{400} = 7.5 \text{ cm}$$

Extension of the spring when it is stretched to equilibrium line is  $x'$ .

$$mg = kx'$$

$$\Rightarrow x' = \frac{2 \times 10}{400} = 5 \text{ cm}$$

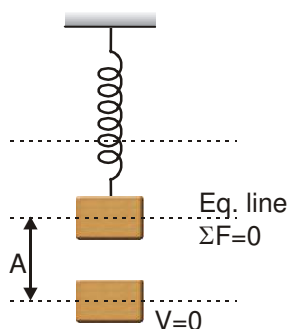
Therefore amplitude  $A = x - x' = 2.5 \text{ cm}$

If upward direction is taken as positive at  $t = 0$ ,  $x = -A$

Using  $x = A \sin(\omega t + \phi)$

$$-A = A \sin \phi$$

$$\phi = \frac{3\pi}{2}$$





## हल: 1 to 3

स्प्रिंग की प्राकृतिक स्थिति से महत्तम खिंचाव  $x$  है।

$$\text{तो } mg + ma = kx$$

$$\Rightarrow x = \frac{2(10+5)}{400} = 7.5 \text{ cm}$$

साम्यवस्था से स्प्रिंग खिंचने पर इसमें खिंचाव  $x'$  है।

$$mg = kx'$$

$$\Rightarrow x' = \frac{2 \times 10}{400} = 5 \text{ cm}$$

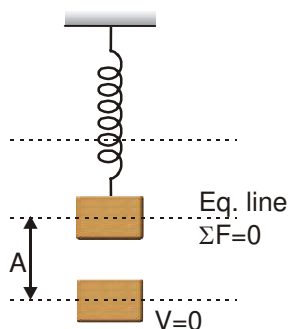
अतः आयाम  $A = x - x' = 2.5 \text{ cm}$

यदि ऊपर की दिशा को धनात्मक लें, तो  $t = 0$  पर  $x = -A$

$x = A \sin(\omega t + \phi)$  का प्रयोग करने पर

$$-A = A \sin \phi$$

$$\phi = \frac{3\pi}{2}$$



## Comprehension # 2

A particle of mass ' $m$ ' moves on a horizontal smooth line AB of length ' $a$ ' such that when particle is at any general point P on the line two forces act on it. A force  $\frac{mg(AP)}{a}$  towards A and another force

$\frac{2mg(BP)}{a}$  towards B.

एक ' $m$ ' द्रव्यमान का कण क्षैतिज चिकनी रेखा AB, जिसकी लम्बाई ' $a$ ' है, पर गति कर रहा है। जब कण रेखा के किसी बिन्दु P पर होता है तो उस पर दो बल कार्य करते हैं। A की ओर बल  $\frac{mg(AP)}{a}$  तथा B की ओर दूसरा बल  $\frac{2mg(BP)}{a}$  कार्यरत है।

4. Find its time period when released from rest from mid-point of line AB.

जब इस कण को रेखा AB के मध्य विरामावस्था से छोड़ा जाता है तो इस कण का आवर्तकाल ज्ञात करो .

(A)  $T = 2\pi\sqrt{\frac{3a}{g}}$       (B)  $T = 2\pi\sqrt{\frac{a}{2g}}$       (C)  $T = 2\pi\sqrt{\frac{a}{g}}$       (D\*)  $T = 2\pi\sqrt{\frac{a}{3g}}$

5. Find the minimum distance of the particle from B during the motion.

गति के दौरान, कण की B से न्यूनतम दूरी ज्ञात करो।

(A\*)  $\frac{a}{6}$       (B)  $\frac{a}{4}$       (C)  $\frac{a}{3}$       (D)  $\frac{a}{8}$





6. If the force acting towards A stops acting when the particle is nearest to B then find the velocity with which it crosses point B.

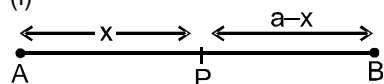
जब कण B के सबसे निकट होता है तो कण पर A की ओर लगने वाला बल कार्य करना बन्द कर देता है तो जिस वेग से यह बिन्दु B को पार करेगा वह वेग ज्ञात करो ?

- (A)  $\frac{\sqrt{2ga}}{3}$  (B\*)  $\frac{\sqrt{2ga}}{6}$  (C)  $\frac{\sqrt{2ga}}{5}$  (D)  $\frac{\sqrt{ga}}{3}$

**Ans.**  $T = 2\pi\sqrt{\frac{a}{3g}}$ ,  $A = \frac{a}{6}$ ,  $\frac{a}{6}$ ,  $1/6\sqrt{2ag}$

**Sol.**

(i)



force on particle at point P

बिन्दु P पर स्थित कण पर बल

$$F = \frac{2mg(a-x)}{a} - \frac{mg(x)}{a}$$

$$F = \frac{mg}{a} (2a - 3x)$$

$$F = \frac{-3mg}{a} (x - 2a/3)$$

(ii) So this is equation of S.H.M ( $F = m\omega^2x$ ) so particle perform S.H.M with mean position  $x - 2a/3 = 0$

अतः ये समीकरण स.आ.ग. की समीकरण है। ( $F = m\omega^2x$ ) अतः कण स.आ.ग. करेगा जिसकी माध्य अवस्था है  $x - 2a/3 = 0$   $x = 2a/3$  (from point A) बिन्दु A से

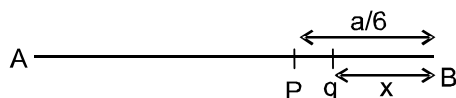
So अतः  $\omega^2 = \frac{3g}{a} \Rightarrow \omega = \sqrt{3g/a}$

So Time period अतः आवर्तकाल  $T = 2\pi\sqrt{a/3g}$

and amplitude और आयाम  $= 2a/3 - a/2 = a/6$

(iii) minimum distance from B से न्यूनतम दूरी  $= a - (2a/3 + a/6) = a/6$

(iv)



at point P velocity of particle = 0

बिन्दु P पर कण का वेग = 0

and force और बल  $= \frac{2mg}{a} x$

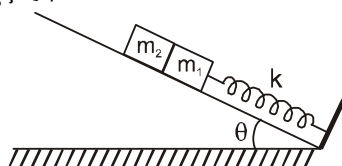
(at point q) (q बिन्दु पर)

$$\text{acc. त्वरण} = \frac{2g}{a} x \Rightarrow \int_0^v v dv = \int_0^{a/6} \frac{2gx}{a} dx \Rightarrow v = \frac{\sqrt{2ga}}{6}$$

### Comprehension # 3

Spring of spring constant k is attached with a block of mass  $m_1$  as shown in figure. Another block of mass  $m_2$  is placed against  $m_1$  and both masses lie on smooth incline plane.

k स्प्रिंग नियतांक का स्प्रिंग  $m_1$  द्रव्यमान के पिण्ड से जुड़ा है तथा दूसरा  $m_2$  द्रव्यमान का पिण्ड पहले पिण्ड को दबाते हुए चित्रानुसार चिकने नत तल पर रखे हुए हैं।





7. Find the compression in the spring when the system is in equilibrium.  
जब निकाय साम्यावस्था में है तब स्प्रिंग का सम्पीड़न ज्ञात करो ?

(A)  $\frac{(m_1 + m_2) g \sin \theta}{2k}$  (B\*)  $\frac{(m_1 + m_2) g \sin \theta}{k}$   
(C)  $\frac{(m_1 + m_2) g}{k}$  (D)  $\frac{2(m_1 + m_2) g \sin \theta}{k}$

8. From the equilibrium position the blocks are pushed a further distance  $\frac{2}{k}(m_1 + m_2)g \sin \theta$  against the spring and released. Find the common speed of blocks when they separate.  
साम्यावस्था से पिण्डों को  $(2/k)(m_1 + m_2) g \sin \theta$  दूरी तक ओर ज्यादा दबाकर छोड़ा जाता है। जब यह पृथक्कृत होते हैं उस समय दोनों पिण्डों की उभयनिष्ठ चाल ज्ञात कीजिए।

(A)  $\left( \sqrt{\frac{1}{3k}}(m_1 + m_2) \right) g \sin \theta$  (B)  $\left( \sqrt{\frac{2}{k}}(m_1 + m_2) \right) g \sin \theta$   
(C\*)  $\left( \sqrt{\frac{3}{k}}(m_1 + m_2) \right) g \sin \theta$  (D)  $\left( \sqrt{\frac{1}{k}}(m_1 + m_2) \right) g \sin \theta$

**Sol.** At equilibrium position साम्यावस्था की स्थिति पर : →

$$Kx_0 = m_1 g \sin \theta + m_2 g \sin \theta$$

$$x_0 = \frac{(m_1 + m_2) g \sin \theta}{k}$$

Block will separate when acceleration of block of mass  $m_1$  will be just equal to  $g \sin \theta$

जब  $m_1$  द्रव्यमान के ब्लॉक का त्वरण  $g \sin \theta$  के बराबर होगा ब्लॉक अलग – अलग हो जायेगे।

$$\Rightarrow a = \omega^2 x = g \sin \theta$$

$$\Rightarrow \frac{K}{m_1 + m_2} \times x = g \sin \theta$$

$$\Rightarrow x = \frac{m_1 + m_2}{K} g \sin \theta = x_0$$

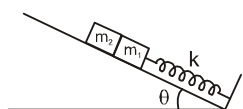
So, blocks will separate when blocks are at distance  $x = \frac{m_1 + m_2}{K} g \sin \theta$

अतः जब ब्लॉक माध्यवस्था से  $x = \frac{m_1 + m_2}{K} g \sin \theta$  की दूरी पर होंगे ब्लॉक अलग अलग हो जायेगे।

From mean position

i.e. spring is at its natural position (amplitude  $A = \left( \frac{2}{K}(m_1 + m_2)g \sin \theta \right)$ )

जब स्प्रिंग इसकी प्राकृतिक स्थिति में है। (आयाम  $A = \left( \frac{2}{K}(m_1 + m_2)g \sin \theta \right)$ )



Blocks will separate at distance  $x = x_0$  from mean position.

माध्यवस्था से  $x = x_0$  दूरी पर ब्लॉक अलग हो जायेगे।

$$v = \sqrt{A^2 - x_0^2}$$

$$= \sqrt{\frac{K}{m_1 + m_2} \left[ \left( \frac{2}{K}(m_1 + m_2)g \sin \theta \right)^2 - \left( \left( \frac{m_1 + m_2}{K} \right) g \sin \theta \right)^2 \right]}$$

$$= \sqrt{\frac{3}{k}}(m_1 + m_2) g \sin \theta$$



## Exercise-3

### PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

#### भाग - I : JEE (ADVANCED) / IIT-JEE (पिछले वर्षों) के प्रश्न

✎ Marked Questions may have for Revision Questions.

✎ चिह्नित प्रश्न दोहराने योग्य प्रश्न है।

\* Marked Questions may have more than one correct option.

\* चिह्नित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है।

1\*. ✎ Function  $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$  represents SHM

$x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$  फलन सरल आवर्त गति प्रदर्शित करेगा।

[JEE 2006, 5/184, -1]

(A\*) for any value of A, B and C (except C = 0) (B\*) If  $A = -B$ ,  $C = 2B$ , amplitude =  $|B\sqrt{2}|$

(C) If  $A = B$ ;  $C = 0$

(D\*) If  $A = B$ ;  $C = 2B$ , amplitude =  $|B|$

(A\*) A, B तथा C के किसी भी मान के लिए (C = 0 को छोड़कर)

(B\*) यदि  $A = -B$ ,  $C = 2B$ , आयाम =  $|B\sqrt{2}|$

(C) यदि  $A = B$ ;  $C = 0$

(D\*) यदि  $A = B$ ;  $C = 2B$ , आयाम =  $|B|$

Ans. (A, B, D)

Sol.  $x = A \frac{(1 - \cos 2\omega t)}{2} + B \frac{(1 + \cos 2\omega t)}{2} + \frac{C}{2} \sin 2\omega t$

Choose different combinations of A, B & C to get linear combination of sin & cos functions.

A, B तथा C के भिन्न-भिन्न मानों के लिये sin व cos फलन के रेखीय मिश्रण प्राप्त करो।

2.# ✎ **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graphs given in **Column II**. [IIT-JEE 2008, 6/163]

#### Column I

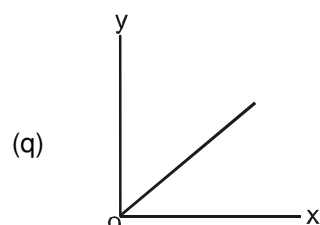
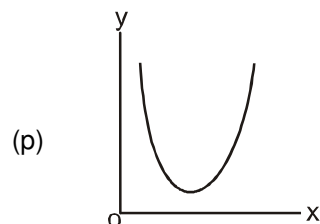
(A) Potential energy of a simple pendulum

(y-axis) as a function of displacement (x-axis)

(B) Displacement (y-axis) as a function of time

(x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.

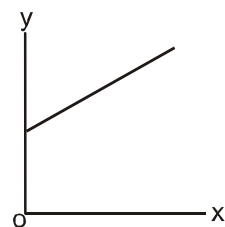
#### Column II





(C) Range of a projectile (y-axis) as a function

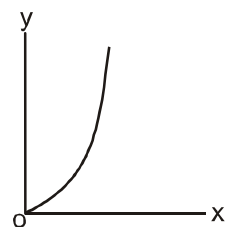
(r)



of its velocity (x-axis) when projected at a fixed angle.

(D) The square of the time period (y-axis) of a

(s)



simple pendulum as a function of its length (x-axis).

**Ans.** (A) → (p) ; (B) → (q, s) ; (C) → (s) ; (D) → (q)

**Sol.** (A) From nature of SHM, the graph of potential energy as function of displacement will be parabolic graph as given in option p. Hence (A) → (p)

(B)  $a = 0$  or  $a = \text{constant}$ . (as per given condition)

$V > 0$  moving along positive x-axis

y – displacement

$$y = ut \pm \frac{1}{2}at^2 \quad \text{for} \quad a = \text{constant}$$

$$y = vt \quad \text{for} \quad a = 0$$

These two conditions are satisfied by (q) and (s).

(B) → (q, s)

(p) is rejected because at  $t = 0$  the displacement is not zero and velocity has negative values.

(C)  $R = \frac{u^2 \sin(2\theta)}{g}$  and  $R \propto u^2$  for a fixed angle of projection. at  $u = 0, R = 0$

(C) → (s)

(D)  $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow y = \frac{4\pi^2}{g} x$  (D) → (q)

**कॉलम I** में संभावी पैरामीटरों (parameters) के समुच्चय (set) की सूची दी गई है। इन पैरामीटरों के परिवर्तन को ग्राफ के रूप में दर्शाया गया है (**कॉलम II**)। **कॉलम I** में दिये गये पैरामीटरों का **कॉलम II** में दिये गये ग्राफ समुल (match) करें। अपने उत्तर को ORS में दिया गया  $4 \times 4$  मैट्रिक्स के उचित बुल्लों को काला करके दर्शायें।

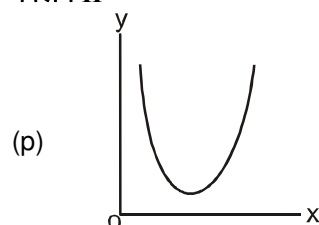
[IIT-JEE 2008, 6/163]

**कॉलम I**

(A) एक सरल लोलक (simple pendulum) की

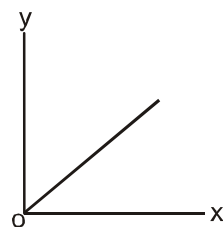
स्थितिज ऊर्जा (y-अक्ष) बनाम विस्थापन (x-अक्ष)

**कॉलम II**



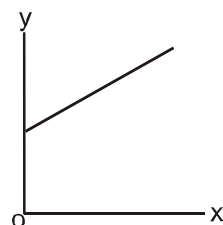


- (B) शून्य अथवा स्थिर त्वरण पर एक विमीय गति के लिये (q)



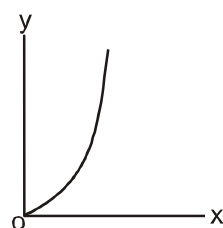
विस्थापन (y-अक्ष) बनाम समय (x-अक्ष) (गति केवल + x दिशा में है)

- (C) एक नियत कोण पर प्रक्षेपित प्रक्षेप्य की परास (r)



(y-अक्ष) बनाम उसका वेग (x-अक्ष)

- (D) एक सरल लोलक के आवर्त काल का वर्ग (y-अक्ष) (s)



बनाम उसकी लम्बाई (x-अक्ष)

**Ans.** (A) → (p) ; (B) → (q, s) ; (C) → (s) ; (D) → (q)

**Sol.** (A) SHM की प्रकृति से, विकल्प p से दिये गए ग्राफ से स्थितिज ऊर्जा-विस्थापन का फलन परवलय होगा  
अतः (A) → (p)

- (B)  $a = 0$  या  $a =$  नियत (दी गई शर्त से)  
 $V > 0$  यह धनात्मक x-अक्ष के अनुदिश है।  
y – विस्थापन

$$y = ut \pm \frac{1}{2}at^2 \quad \text{for} \quad a = \text{नियत}$$

$$y = vt \quad \text{for} \quad a = 0$$

यह दोनों स्थितियां (q) व (s) द्वारा संतुष्ट होती हैं।

(B) → (q, s)

(p) को नहीं लिया गया है चूंकि  $t = 0$  पर विस्थापन शून्य नहीं है तथा वेग का ऋणात्मक मान है।

- (C)  $R = \frac{u^2 \sin(2\theta)}{g}$  and  $R \propto u^2$  एक नियत प्रक्षेपण कोण के लिए

$$\text{तथा } u = 0, R = 0 \Rightarrow (C) \rightarrow (s)$$

- (D)  $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow y = \frac{4\pi^2}{g}x \Rightarrow (D) \rightarrow (q)$



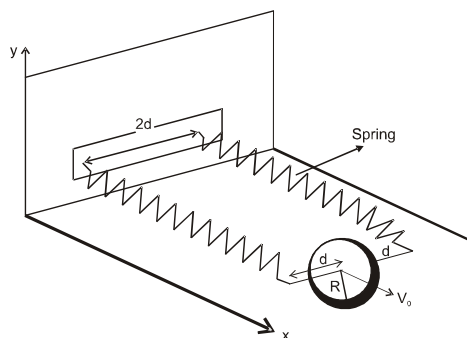
**Comprehension # 1 अनुच्छेद # 1**

A uniform thin cylindrical disk of mass  $M$  and radius  $R$  is attached to two identical massless springs of spring constant  $k$  which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance  $d$  from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is  $L$ . The disk is initially at its equilibrium position with its centre of mass (CM) at a distance  $L$  from the wall. The disk rolls without slipping with velocity  $\vec{V}_0 = V_0 \hat{i}$ . The coefficient of friction is  $\mu$ . [JEE-2008, 3×4/163]

Figure :

चित्रानुसार एकसमान (uniform) पतली बेलनाकार चकती, जिसका द्रव्यमान  $M$  तथा त्रिज्या  $R$  है, दो सर्वसम द्रव्यमान रहित स्प्रिंग (spring) के द्वारा, जिनका स्प्रिंग स्थिरांक  $k$  है, दीवार से जुड़ी है। दोनों स्प्रिंग चकती के केन्द्र से  $d$  की सममित (symmetric) दूरी पर, धुरी से संलग्न हैं। धुरी द्रव्यमान रहित है तथा दोनों स्प्रिंग और धुरी एक क्षैतिज समतल पर हैं। प्रत्येक स्प्रिंग की सामान्य लंबाई  $L$  है। प्रारंभ में चकती का द्रव्यमान केन्द्र साम्यावस्था में दीवार से  $L$  की दूरी पर है। डिस्क बिना फिसले  $\vec{V}_0 = V_0 \hat{i}$  की गति से लुढ़कती है। घर्षण गुणांक  $\mu$  है। [JEE-2008, 3×4/163]

चित्र :



3. The net external force acting on the disk when its centre of mass is at displacement  $x$  with respect to its equilibrium position is

जब चकती का द्रव्यमान केन्द्र (centre of mass) इसकी साम्यावस्था से  $x$  की दूरी पर विस्थापित है, चकती के ऊपर लगने वाला कुल बाहरी बल निम्न है

- (A)  $-kx$  (B)  $-2kx$  (C)  $-\frac{2kx}{3}$  (D\*)  $-\frac{4kx}{3}$

**Sol.** Applying equation of torque about lowest point  
सबसे निम्नतम बिन्दु पर बलाघूर्ण का समीकरण लगाने पर

$$(2Kx) R = \left( \frac{3}{2} MR^2 \right) \alpha \quad \alpha R = \frac{4Kx}{3M}$$

as there is no slipping चूंकि कोई फिसलन नहीं है।

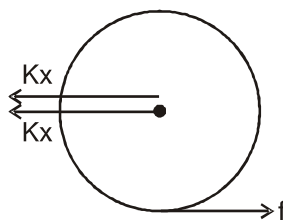
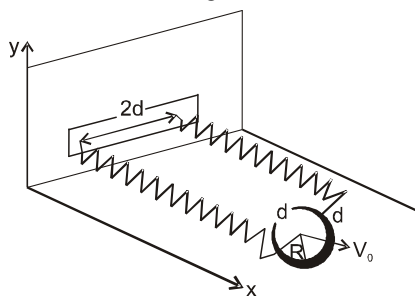
$$a = \alpha R = \frac{4Kx}{3M}$$

$$\text{Net force कुल बल} = Ma = \frac{4Kx}{3}$$

Which is directed opposite to displacement जो कि विस्थापन के विपरीत है।

$$F_{\text{net}} = \frac{-4Kx}{3}$$

**Ans. (D)**





4. The centre of mass of the disk undergoes simple harmonic motion with angular frequency  $\omega$  equal to चकती का द्रव्यमान केन्द्र सरल आवर्त गति करता है, जिसकी कोणीय आवृत्ति  $\omega$  का मान निम्न है

(A)  $\sqrt{\frac{k}{M}}$  (B)  $\sqrt{\frac{2k}{M}}$  (C)  $\sqrt{\frac{2k}{3M}}$  (D\*)  $\sqrt{\frac{4k}{3M}}$

**Sol.**  $F_{\text{net}} = -\frac{4kx}{3} = -M(\omega^2 x)$

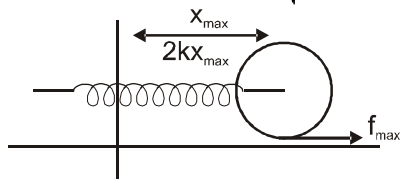
$\omega = \sqrt{\frac{4k}{3M}}$  **Ans. (D).**

5. The maximum value of  $V_0$  for which the disk will roll without slipping is  $V_0$  का वह महत्तम मान जिसके लिये चकती बिना फिसले लुढ़कती है

(A)  $\mu g \sqrt{\frac{M}{k}}$  (B)  $\mu g \sqrt{\frac{M}{2k}}$  (C\*)  $\mu g \sqrt{\frac{3M}{k}}$  (D)  $\mu g \sqrt{\frac{5M}{2k}}$

**Sol.**  $\frac{1}{2} MV_0^2 + \frac{1}{2} \frac{MR^2}{2} \left( \frac{V_0}{R} \right)^2 = \frac{1}{2} (2K) x_{\text{max}}^2$

$= \frac{3}{2} MV_0^2 = 2Kx_{\text{max}}^2 \Rightarrow x_{\text{max}} = \sqrt{\frac{3}{4} \frac{MV_0^2}{K}}$



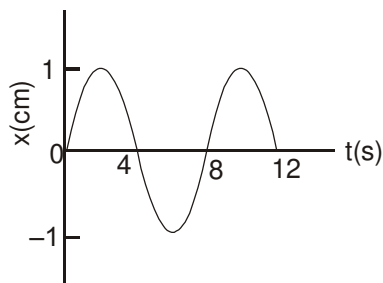
At extreme position, friction will have maximum value.  
चरम स्थिति पर, घर्षण अधिकतम होगा।

$2Kx_{\text{max}} - f_{\text{max}} = \frac{4k}{3} x_{\text{max}} \Rightarrow f_{\text{max}} = \frac{2}{3} Kx_{\text{max}}$

$\mu Mg = \frac{2}{3} K \sqrt{\frac{3}{4} \frac{MV_0^2}{K}} \Rightarrow \mu Mg = \left( \sqrt{\frac{K}{3}} M \right) V_0$

$V_0 = \mu g \sqrt{\frac{3M}{K}}$  **Ans. (C).**

6. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at  $t = 4/3$  s is [IIT-JEE 2009, 3/160, -1]  
सरल आवर्त गति करते हुए किसी कण का x-t आरेख नीचे दर्शाया गया है। समय  $t = 4/3$  सैकेण्ड पर कण का त्वरण है



(A)  $\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$  (B)  $-\frac{\pi^2}{32} \text{ cm/s}^2$  (C)  $\frac{\pi^2}{32} \text{ cm/s}^2$  (D\*)  $\frac{\sqrt{3}}{32} \pi^2 - \text{cm/s}^2$



**Solution :** From graph (ग्राफ से)

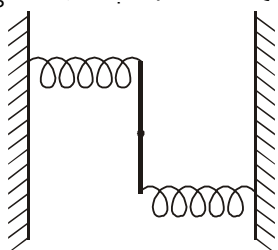
$$T = 8 \text{ second.}, \quad A = 1 \text{ cm}, \quad x = A \sin \omega t. = 1 \sin \frac{2\pi}{8} t.$$

$$a = -\omega^2 x = -\left(\frac{2\pi}{8}\right)^2 \sin\left(\frac{2\pi}{8}\right) t \text{ cm/s}^2$$

$$\text{At, } t = \frac{4}{3} \text{ second पर } a = \left(\frac{2\pi}{8}\right)^2 - \frac{\pi}{3} \sin = -\frac{\sqrt{3}}{32} \pi^2 \text{ cm/s}^2$$

- 7.#** A uniform rod of length  $L$  and mass  $M$  is pivoted at the centre. Its two ends are attached to two springs of equal spring constants  $k$ . The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle  $\theta$  in one direction and released. The frequency of oscillation is : **[IIT-JEE 2009, 3/160, -1]**

चित्रानुसार लम्बाई  $L$  व द्रव्यमान  $M$  की एकसमान छड़ अपने केन्द्र पर कीलकित है। इस छड़ के सिरों पर  $k$  स्प्रिंग नियतांक के एक जैसे स्प्रिंग लगे हैं जिनके सिरे दृढ़ आलम्बों से जुड़े हैं। छड़ क्षैतिज तल में स्वतन्त्र रूप से दोलन कर सकती है। छड़ को एक छोटे कोण  $\theta$  से घुमा कर छोड़ दिया जाता है। छड़ के दोलन की आवृत्ति होगी।



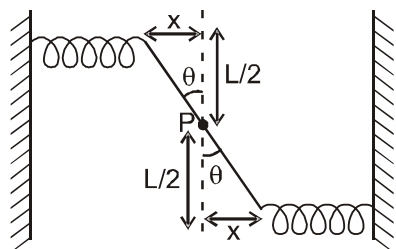
(A)  $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$

(B)  $\frac{1}{2\pi} \sqrt{\frac{k}{M}}$

(C\*)  $\frac{1}{2\pi} \sqrt{\frac{6k}{M}}$

(D)  $\frac{1}{2\pi} \sqrt{\frac{24k}{M}}$

**Sol.**



$$\text{Torque about P} = (kx) \frac{L}{2} + (kx) \frac{L}{2} = kxL \quad (\theta = \frac{2x}{L})$$

$$\tau = I \alpha$$

$$\Rightarrow -\frac{KL^2}{2} \theta = \frac{ML^2}{12} \alpha \Rightarrow \frac{-6K\theta}{M} = \alpha \Rightarrow \alpha = -\frac{6K}{M} \theta = -\omega^2 \theta$$

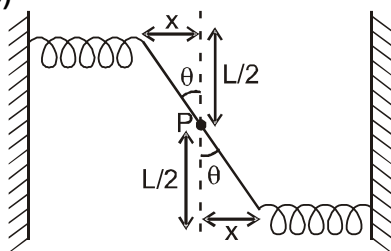
$$\Rightarrow \omega = \sqrt{\frac{6K}{M}} \text{ and } f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{6K}{M}}$$







Sol. (C)



$$P \text{ के सापेक्ष बल आघूर्ण} = (kx) \frac{L}{2} + (kx) \frac{L}{2} = kxL \quad (\theta = \frac{2x}{L})$$

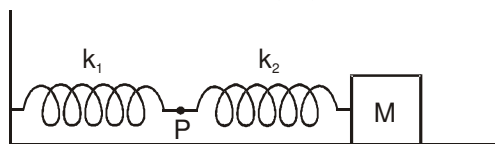
$$\tau = I \alpha$$

$$\Rightarrow -\frac{KL^2}{2} \theta = \frac{ML^2}{12} \alpha \Rightarrow \frac{-6K\theta}{M} = \alpha \Rightarrow \alpha = -\frac{6K}{M} \theta = -\omega^2 \theta$$

$$\Rightarrow \omega = \sqrt{\frac{6K}{M}} \text{ और } f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{6K}{M}}$$

8.# The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is [JEE 2009, 3/160, -1]

चित्र में दिखाया गया द्रव्यमान M सरल आवर्त गति कर रहा है जिसका आयाम A है। बिन्दु P का आयाम होगा



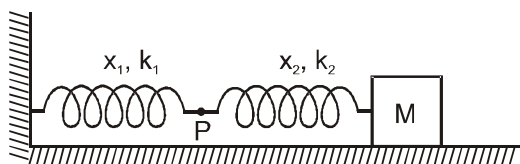
(A)  $\frac{k_1 A}{k_2}$

(B)  $\frac{k_2 A}{k_1}$

(C)  $\frac{k_1 A}{k_1 + k_2}$

(D\*)  $\frac{k_2 A}{k_1 + k_2}$

Sol.



Extensions in springs are  $x_1$  and  $x_2$  then

स्प्रिंगों में विस्तार  $x_1$  व  $x_2$  है तब

$$k_1 x_1 = k_2 x_2$$

$$\text{and और } x_1 + x_2 = A$$

$$\Rightarrow x_1 + \frac{k_1 x_1}{k_2} = A \Rightarrow x_1 = \frac{k_2 A}{k_1 + k_2}$$

### Comprehension # 2 अनुच्छेद # 2

When a particle of mass m moves on the x-axis in a potential of the form  $V(x) = kx^2$ , it performs simple harmonic motion. The corresponding time period is proportional to  $\sqrt{\frac{m}{k}}$ , as can be seen easily using

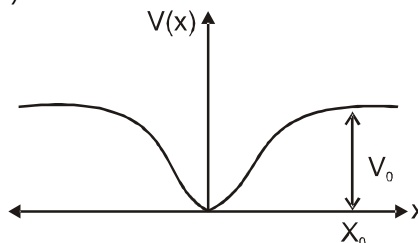
dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of  $x = 0$  in a way different from  $kx^2$  and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is  $V(x) = \alpha x^4$  ( $\alpha > 0$ ) for  $|x|$  near the origin and becomes a constant equal to  $V_0$  for  $|x| \geq X_0$  (see figure)

[JEE 2010, 9/160, -1]



जब  $m$  द्रव्यमान का एक कण  $x$ -अक्ष पर  $V(x) = kx^2$  स्थितिज ऊर्जा से गतिमान होता है, तब यह सरल आवर्त गति करता है। इसका आवर्तकाल  $\sqrt{\frac{m}{k}}$  के समानुपाती होता है, जो कि विमीय विश्लेषण द्वारा आसानी से निकाला जा सकता है। हालांकि, यदि किसी एक कण की स्थितिज ऊर्जा  $x = 0$  के दोनों तरफ  $kx^2$  से भिन्न तरह से बढ़े, तथा कण की कुल ऊर्जा इतनी हो कि वह अनन्त तक पलायन न कर सके, तब भी कण की गति आवर्ती हो सकती है।  $m$  द्रव्यमान का एक कण  $x$ -अक्ष पर गति करता है जहाँ  $|x|$  के केन्द्र के पास होने पर स्थितिज ऊर्जा  $V(x) = \alpha x^4$  ( $\alpha > 0$ ) है तथा  $|x| \geq X_0$  के लिए स्थितिज ऊर्जा  $V(x) = V_0$  है (चित्र देखें)

[JEE 2010, 9/160, -1]



9. If the total energy of the particle is  $E$ , it will perform periodic motion only if :  
यदि एक कण की समग्र ऊर्जा  $E$  है तो वह आवर्ती गति तभी सम्पन्न कर सकता है केवल जब  
(A)  $E < 0$  (B)  $E > 0$  (C\*)  $V_0 > E > 0$  (D)  $E > V_0$
- Sol. When  $0 < E < V_0$  there will be acting a restoring force to perform oscillation because in this case particle will be in the region  $|x| \leq x_0$ .
- Sol. जब  $0 < E < V_0$ , यहाँ दोलन करने के लिए एक प्रत्यानयन बल कार्यरत है क्योंकि इस स्थिति में कण क्षेत्र  $|x| \leq x_0$  में रहता है।

10. For periodic motion of small amplitude  $A$ , the time period  $T$  of this particle is proportional to :  
अल्प आयाम  $A$  के दोलन के लिए, कण का आवर्तकाल  $T$  निम्न में से किसके समानुपाती है ?

(A)  $A\sqrt{\frac{m}{\alpha}}$  (B\*)  $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$  (C)  $A\sqrt{\frac{\alpha}{m}}$  (D)  $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

- Sol.  $V = \alpha x^4$   
T.E. =  $\frac{1}{2} m \omega^2 A^2 = \alpha A^4$  (not strictly applicable just for dimension matching it is used)  
T.E. =  $\frac{1}{2} m \omega^2 A^2 = \alpha A^4$  (पूर्णतः लागू नहीं है केवल यह विमीय रूप से मिलान के लिए उपयोगी है)  
 $\omega^2 = \frac{2\alpha A^2}{m} \Rightarrow T \propto \frac{1}{A}\sqrt{\frac{m}{\alpha}}$

11. The acceleration of this particle for  $|x| > X_0$  is :  $|x| > X_0$  के लिये कण का त्वरण :

(A) proportional to  $V_0$  (B) proportional to  $\frac{V_0}{mX_0}$   
(C) proportional to  $\sqrt{\frac{V_0}{mX_0}}$  (D\*) zero  
(A)  $V_0$  के समानुपाती है (B)  $\frac{V_0}{mX_0}$  के समानुपाती है  
(C)  $\sqrt{\frac{V_0}{mX_0}}$  के समानुपाती है (D\*) शून्य है



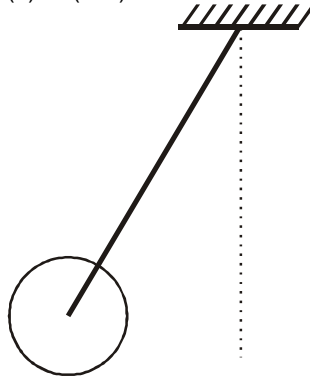
**Sol.**  $F = -\frac{dU}{dx}$

as for  $|x| > x_0$   $V = V_0 = \text{constant}$

जैसा कि  $|x| > x_0$  के लिए  $V = V_0 = \text{नियतांक}$

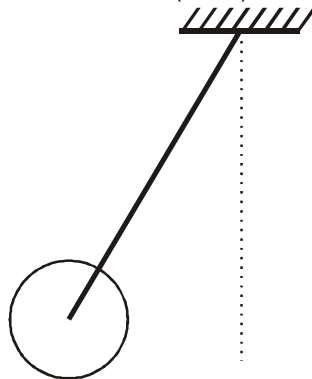
$$\Rightarrow \frac{dU}{dx} = 0 \Rightarrow F = 0$$

- 12\*.** A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' ( $< L$ ) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? [JEE 2011, 4/160]



- (A\*) Restoring torque in case A = Restoring torque in case B  
 (B) Restoring torque in case A  $<$  Restoring torque in case B  
 (C) Angular frequency for case A  $>$  Angular frequency for case B.  
 (D\*) Angular frequency for case A  $<$  Angular frequency for case B.

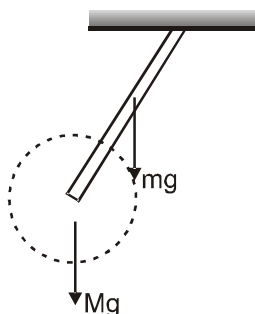
लम्बाई 'L' व द्रव्यमान 'm' की एक धातु-छड़ अपने एक सिरे पर कीलित है। 'R' ( $< L$ ) त्रिज्या व 'M' द्रव्यमान की एक डिस्क छड़ के मुक्त सिरे पर अपने केंद्र से दो तरह से कीलित की जाती है। (स्थिति A) डिस्क अपने केंद्र पर नहीं घूम सकती और (स्थिति B) डिस्क अपने केंद्र पर घूमने को स्वतन्त्र है। एक ही विस्थापन अवस्था से छोड़ने पर यह छड़-डिस्क निकाय उर्ध्वाधर तल में सरल-आवर्त-दोलन (SHM) करता है। तब



- (A\*) स्थिति A में प्रत्यानयन आघूर्ण (Restoring torque) = स्थिति B में प्रत्यानयन आघूर्ण  
 (B) स्थिति A में प्रत्यानयन आघूर्ण  $<$  स्थिति B में प्रत्यानयन आघूर्ण  
 (C) स्थिति A में कोणीय आवृत्ति  $>$  स्थिति B में कोणीय आवृत्ति  
 (D\*) स्थिति A में कोणीय आवृत्ति  $<$  स्थिति B में कोणीय आवृत्ति



Ans. (A), (D)



Sol. torque is same for both the cases.  
दोनों प्रकरणों में बलाघूर्ण समान है

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

$$I_A > I_B$$

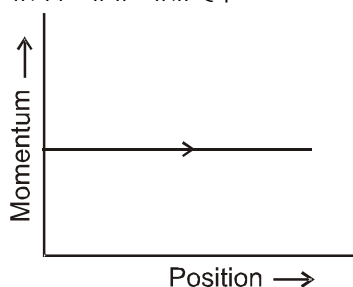
$$\omega_A < \omega_B$$

### Comprehension # 3 अनुच्छेद # 3

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is  $x(t)$  vs.  $p(t)$  curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum, upwards (or to right) is positive and downwards (or to left) is negative.

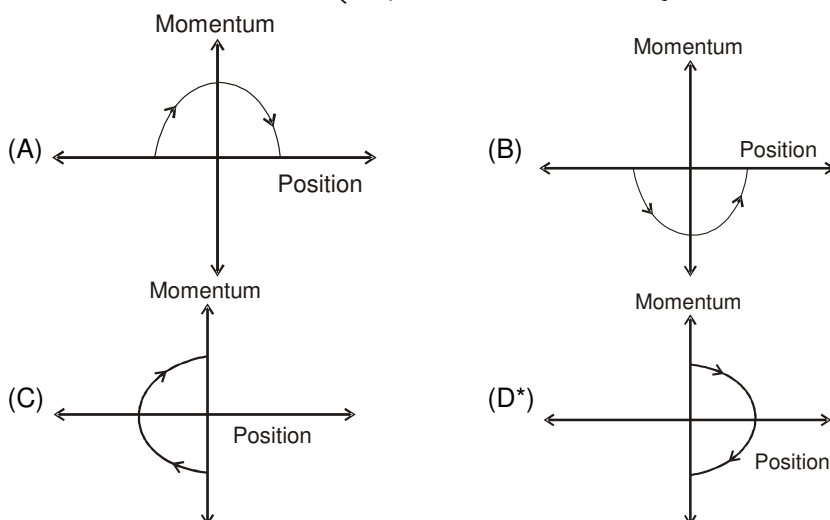
[JEE 2011, 3×3/160, -1]

हर प्रकार की गतिकीय समस्याओं के विश्लेषण के लिये फेज-समष्टि चित्राम (Phase space diagrams) का उपयोग किया जाता है। प्रारम्भिक दशा, स्थिति व संवेग, में बदलाव होने पर इनका उपयोग चालन में उत्पन्न बदलावों को समझने में बहुत उपयोगी है। यहाँ हम एक विमीय सरल गतिकीय निकायों की बात करते हैं। इनके लिये फेज-समष्टि समतल है जिसमें स्थिति X-अक्ष पर तथा संवेग Y-अक्ष पर रखते हैं। तब फेज-समष्टि चित्राम इस समतल में एक  $x(t)$  vs.  $p(t)$  वक्र होगा। वक्र पर तीर समय बढ़ने की दिशा दर्शाता है। उदाहरण के लिये, स्थिर वेग से चल रहे कण के लिये फेज-समष्टि चित्राम सरल-रेखा है जिसे चित्र में दिखाया गया है। चिन्ह परिपाटी में स्थिति या संवेग को ऊपर (या दाहिने) ओर धनात्मक तथा नीचे (या बाँयी) ओर ऋणात्मक माना जाता है।

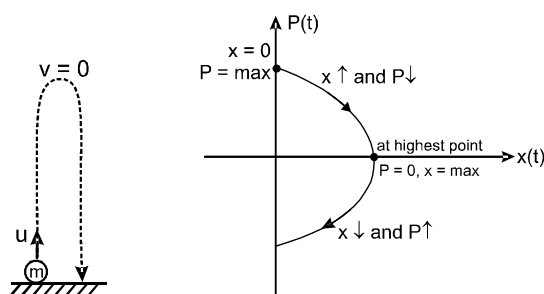




- 13.# The phase space diagram for a ball thrown vertically up from ground is :  
जमीन से ऊपर की ओर फेंकी गई गेंद का फेज-समष्टि चित्राम है

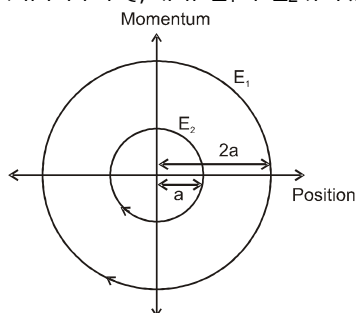


Sol.



- 14.# The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and  $E_1$  and  $E_2$  are the total mechanical energies respectively. Then

सरल-आवर्त-दोलन (SHM) का फेज-समष्टि चित्राम उदगम पर केंद्रित वृत्त है। चित्र में दर्शाये दो वृत्त, एक ही दोलक के लिये है, जब उसकी आरम्भिक अवस्थायें भिन्न हैं, तथा  $E_1$  व  $E_2$  क्रमशः दोलक की कुल यांत्रिक ऊर्जाएँ हैं। तब



- (A)  $E_1 = \sqrt{2} E_2$  (B)  $E_1 = 2 E_2$  (C\*)  $E_1 = 4 E_2$  (D)  $E_1 = 16 E_2$

Ans. (C)

Sol.

In 1st case amplitude of SHM is  $a$ .  
In 2nd case amplitude of SHM is  $2a$

$$\text{Total energy} = \frac{1}{2} k(\text{amplitude})^2$$

$$E_1 = \frac{1}{2} k(2a)^2$$

$$E_2 = \frac{1}{2} k(a)^2 \quad E_1 = 4 E_2$$

**Alternative :**Linear momentum  $P = mv$ 

$$= m\omega \sqrt{A^2 - x^2}$$

$$\Rightarrow P^2 = m^2\omega^2 (A^2 - x^2)$$

$$\Rightarrow P^2 + (m\omega)^2 x^2 = m^2\omega^2 A^2 \quad \dots(i)$$

Equation of circle (bigger)

$$P^2 + x^2 = (2a)^2$$

$$P^2 + x^2 = 4a^2 \quad \dots(ii)$$

Equation of circle (smaller)

$$P^2 + x^2 = a^2 \quad \dots(iii)$$

Comparing (i) and (ii)

Amplitude  $A = 2a$ 

$$\text{and } (m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$$

$$\frac{1}{2} m\omega^2 (A)^2$$

$$\text{So energy } E_1 = \frac{1}{2} m\omega^2 (2a)^2$$

$$= \frac{1}{2} \times \frac{1}{m} \times (4a^2)$$

$$= \frac{2a^2}{m}$$

Comparing (i) and (iii)

 $A = a$ 

$$(m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$$

$$\text{So } E_2 = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \times \frac{1}{m} a^2 = \frac{1}{2} \frac{a^2}{m}$$

$$\text{So } \frac{E_1}{E_2} = 4 \Rightarrow E_1 = 4E_2$$

**Sol.**प्रथम स्थिति में SHM का आयाम  $a$  है।द्वितीय स्थिति में SHM का आयाम  $2a$  है।

$$\text{कुल ऊर्जा} = \frac{1}{2} k(\text{आयाम})^2$$

$$E_1 = \frac{1}{2} k(2a)^2 \Rightarrow E_2 = \frac{1}{2} k(a)^2$$

$$E_1 = 4 E_2$$

**वैकल्पिक :**रेखीय संवेग  $P = mv$ 

$$= m\omega \sqrt{A^2 - x^2}$$

$$\Rightarrow P^2 = m^2\omega^2 (A^2 - x^2)$$

$$\Rightarrow P^2 + (m\omega)^2 x^2 = m^2\omega^2 A^2 \quad \dots(i)$$

वृत्त (बड़ा वाला) का समीकरण

$$P^2 + x^2 = (2a)^2$$

$$P^2 + x^2 = 4a^2 \quad \dots(ii)$$

वृत्त (छोटा वाला) का समीकरण

$$P^2 + x^2 = a^2 \quad \dots(iii)$$

(i) तथा (ii) की तुलना करने पर

आयाम  $A = 2a$ 
**Resonance®**  
 Educating for better tomorrow

Corporate Office : CG Tower, A-46 &amp; 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)

Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 12



$$\text{तथा } (m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$$

$$\frac{1}{2}m\omega^2(A)^2$$

$$\text{अतः ऊर्जा } E_1 = \frac{1}{2}m\omega^2(2a)^2$$

$$= \frac{1}{2} \frac{1}{m} \times (4a^2)$$

$$= \frac{2a^2}{m}$$

(i) तथा (iii) की तुलना करने पर

$$A = a$$

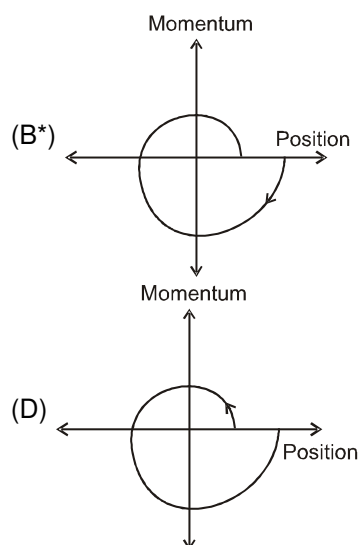
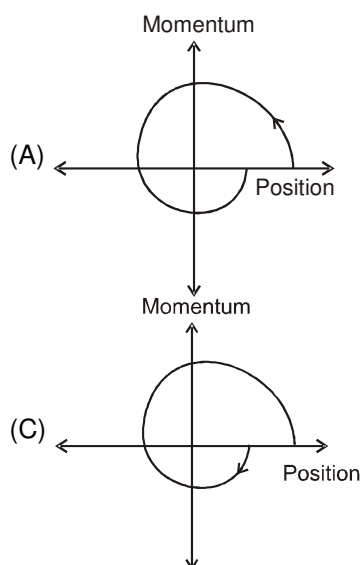
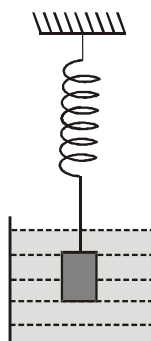
$$(m\omega)^2 = 1 \Rightarrow m\omega^2 = \frac{1}{m}$$

$$\text{अतः } E_2 = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times \frac{1}{m} a^2 = \frac{1}{2} \frac{a^2}{m}$$

$$\text{अतः } \frac{E_1}{E_2} = 4 \Rightarrow E_1 = 4E_2$$

**15.#** Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is :

चित्र में दर्शाये अनुसार स्प्रिंग-गुटका निकाय पर ध्यान दे, जहाँ गुटका पानी में डूबा है। इस निकाय के एक दोलन करने का फेज-समष्टि चित्रांश है।



S.H.M.

Ans. (B)



**Resonance®**  
Educating for better tomorrow

Corporate Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : [www.resonance.ac.in](http://www.resonance.ac.in) | E-mail : [contact@resonance.ac.in](mailto:contact@resonance.ac.in)

Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 13



**Sol.** Linear momentum

$$P = mv$$

$$= m\omega\sqrt{A^2 - x^2}$$

$$\Rightarrow P^2 + m^2\omega^2x^2 = m^2\omega^2A^2$$

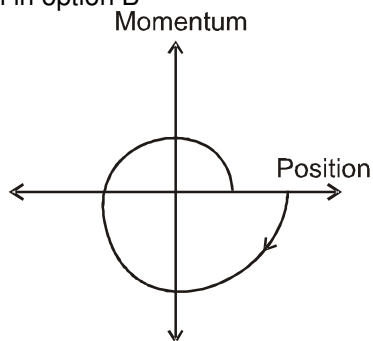
represents a circle on P-x diagram with radius of circle  $R = A$  ( $\because m^2\omega^2 = 1$ )

$\omega$  of spring mass system remains constant and equal to  $\sqrt{\frac{k}{m}}$

Amplitude of oscillation inside liquid will decrease due to viscous force

So radius of circular arcs will decrease as position change

Correctly shown in option B



रेखीय संवेग

$$P = mv = m\omega\sqrt{A^2 - x^2}$$

$$\Rightarrow P^2 + m^2\omega^2x^2 = m^2\omega^2A^2$$

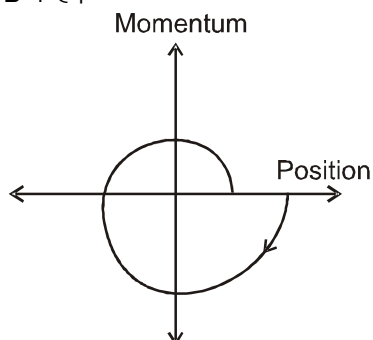
$R = A$  त्रिज्या के साथ P-x चित्र पर वृत्त को प्रदर्शित करता है ( $\because m^2\omega^2 = 1$ )

स्प्रिंग द्रव्यमान निकाय का  $\omega$  नियत रहता है तथा  $\sqrt{\frac{k}{m}}$  के बराबर होता है

द्रव के अन्दर दोलन का आयाम श्यान बल के कारण घटेगा

अतः वृत्ताकार चाप की त्रिज्या स्थिति परिवर्तन के अनुसार घटेगी

सही ग्राफ विकल्प B में है।



16. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction,  $x_1(t) = A \sin \omega t$  and  $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$ . Adding a third sinusoidal displacement  $x_3(t) = B \sin(\omega t + \phi)$  brings the mass to a complete rest. The values of B and  $\phi$  are

[JEE 2011, 3/160, -1]

एक बिन्दु द्रव्यमान पर दो ज्यावक्रीय (sinusoidal) विस्थापन  $x_1(t) = A \sin \omega t$  एवं  $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$  एक साथ x-दिशा में लग रहे हैं। इस द्रव्यमान पर तीसरा ज्यावक्रीय विस्थापन  $x_3(t) = B \sin(\omega t + \phi)$  लगाने पर वह पूर्ण रूप से रुक जाता है। B तथा  $\phi$  का मान है

[JEE 2011, 3/160, -1]

(A)  $\sqrt{2}A, \frac{3\pi}{4}$

(B\*)  $A, \frac{4\pi}{3}$

(C)  $\sqrt{3} A, \frac{5\pi}{6}$

(D)  $A, \frac{\pi}{3}$

Ans. (B)



**Resonance**  
Educating for better tomorrow

Corporate Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in

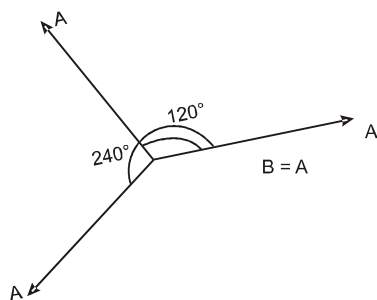
Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 14





Sol.



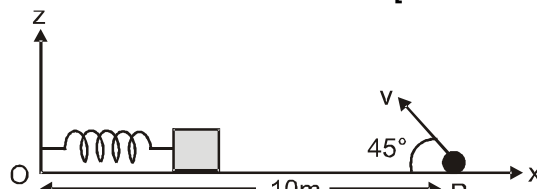
So अतः  $B = A$ ,  $\phi = 240^\circ = \frac{4\pi}{3}$

17. # A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at  $t = 0$ . It then executes simple harmonic motion with angular frequency  $\omega = \frac{\pi}{3} \text{ rad/s}$ . Simultaneously at  $t = 0$ , a small pebble is projected with speed  $v$  from

point P at an angle of  $45^\circ$  as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at  $t = 1 \text{ s}$ , the value of  $v$  is (take  $g = 10 \text{ m/s}^2$ )

एक द्रव्यमान-रहित स्प्रिंग की तनाव-रहित लम्बाई 4.9 cm है। उसका एक सिरा बंधित है और दूसरे पर एक छोटा गुटका लगा है (चित्र देखिये)। यह निकाय एक घर्षण-रहित क्षैतिज (horizontal) सतह पर रखा है। समय  $t = 0$  पर गुटके को 0.2 m खींच कर स्थिर अवस्था से छोड़ा जाता तब वह गुटका  $\omega = \frac{\pi}{3} \text{ rad/s}$  आवृत्ति का सरल-आवर्त-दोलन करता है। ठीक उसी समय ( $t = 0$ ) पर छोटा कंकड़  $v$  चाल से क्षैतिज से  $45^\circ$  कोण पर बिंदु P से प्रक्षेपित किया जाता है। बिंदु P की बिंदु O से दूरी (क्षैतिज) 10 m है। यदि  $t = 1 \text{ s}$  पर कंकड़ गुटके पर गिरता है, तब  $v$  का मान है ( $g = 10 \text{ m/s}^2$  लें)

[IIT-JEE-2012, Paper-1; 3/70, -1]

(A\*)  $\sqrt{50} \text{ m/s}$ (B)  $\sqrt{51} \text{ m/s}$ (C)  $\sqrt{52} \text{ m/s}$ (D)  $\sqrt{53} \text{ m/s}$ 

Sol. Time of flight for projectile प्रक्षेप के लिये उड़ान काल

$$T = \frac{2u \sin \theta}{g} = 1 \text{ sec.}$$

$$\frac{2u \sin 45^\circ}{g} = 1 \text{ sec.}$$

$$u = \frac{g}{\sqrt{2}}$$

$$u = \sqrt{50} \text{ m/s}$$



- 18\*. A particle of mass  $m$  is attached to one end of a mass-less spring of force constant  $k$ , lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time  $t = 0$  with an initial velocity  $u_0$ . When the speed of the particle is  $0.5 u_0$ , it collides elastically with a rigid wall. After this collision : **[JEE (Advanced) 2013, 4/60]**

(A\*) the speed of the particle when it returns to its equilibrium position is  $u_0$ .

(B) the time at which the particle passes through the equilibrium position for the first time is  $t = \pi \sqrt{\frac{m}{k}}$ .

(C) the time at which the maximum compression of the spring occurs is  $t = \frac{4\pi}{3} \sqrt{\frac{m}{k}}$ .

(D\*) the time at which the particle passes throughout the equilibrium position for the second time is  $t = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$ .

घर्षणहीन क्षैतिज तल पर पड़ी हुई  $k$  बल स्थिरांक की द्रव्यमान रहित स्प्रिंग के एक सिरे से  $m$  द्रव्यमान का कण जुड़ा हुआ है। इस स्प्रिंग का दूसरा सिरा बद्ध है। यह कण अपनी साम्यावस्था से समय  $t = 0$  पर प्रारम्भिक क्षैतिज वेग  $u_0$  से गतिमान हो रहा है। जब कण की गति  $0.5 u_0$  होती है, यह एक दृढ़ दीवार से प्रत्यास्थ संघट्ट करता है। इस संघट्ट के बाद -

(A) जब कण अपनी साम्यावस्था से लौटता है इसकी गति  $u_0$  होती है।

(B) जब कण अपनी साम्यावस्था से पहली बार गुजरता है वह समय  $t = \pi \sqrt{\frac{m}{k}}$  है।

(C) जब स्प्रिंग से सम्पीड़न अधिकतम होता है वह समय  $t = \frac{4\pi}{3} \sqrt{\frac{m}{k}}$  है।

(D) जब कण अपनी साम्यावस्था से दूसरी बार गुजरता है वह समय  $t = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$  है।

Ans. (A,D)

Sol  $x = A \sin \omega t$

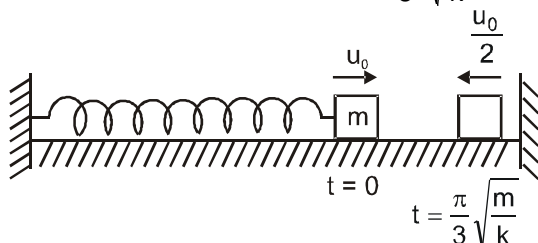
$$v = A \omega \cos \omega t = \frac{\omega A}{2}$$

$$\Rightarrow \cos \omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{3} \Rightarrow t = \frac{2\pi}{3} = \frac{\pi}{3} \sqrt{\frac{m}{k}}$$

$$\text{for (C) time} = \frac{2\pi}{3} \sqrt{\frac{m}{k}} + \frac{\pi}{2} \sqrt{\frac{m}{k}} = \frac{5\pi}{6} \sqrt{\frac{m}{k}}$$

$$\text{for (D) time} = \frac{2\pi}{3} \sqrt{\frac{m}{k}} + \pi \sqrt{\frac{m}{k}} = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$$





**Hindi**  $x = A \sin \omega t$

$$v = A\omega \cos \omega t = \frac{\omega A}{2}$$

$$\Rightarrow \cos \omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{3} \Rightarrow t = \frac{2\pi}{3} = \frac{\pi}{3} \sqrt{\frac{m}{k}}$$

$$(C) \text{ के लिए } \text{समय} = \frac{2\pi}{3} \sqrt{\frac{m}{k}} + \frac{\pi}{2} \sqrt{\frac{m}{k}} \\ = \frac{5\pi}{6} \sqrt{\frac{m}{k}}$$

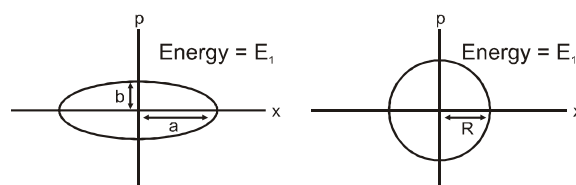
$$(D) \text{ के लिए } \text{समय} = \frac{2\pi}{3} \sqrt{\frac{m}{k}} + \pi \sqrt{\frac{m}{k}} \\ = \frac{5\pi}{3} \sqrt{\frac{m}{k}}$$

**19. #** Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies  $\omega_1$  and  $\omega_2$  and have total energies  $E_1$  and  $E_2$ , respectively. The variations of their momenta  $p$  with positions  $x$  are shown in figures. If  $\frac{a}{b} = n^2$  and  $\frac{a}{R} = n$ , then the correct equation(s) is (are) :

दो निरवलंबित बराबर द्रव्यमान के आवर्त दोलक मूल बिन्दु के परितः कोणीय आवृत्तियों  $\omega_1$  एवं  $\omega_2$  तथा कुल ऊर्जाओं  $E_1$  तथा  $E_2$  से दोलन कर रहे हैं। उनके सर्वेगों  $p$  के स्थिति  $x$  के साथ परिवर्तन संबंध चित्रों में दर्शाये गये हैं। यदि

$\frac{a}{b} = n^2$  तथा  $\frac{a}{R} = n$  है, तब सही कथन है (हैं)

[JEE(Advanced) 2015 ; P-1, 4/88, -2]



(A)  $E_1 \omega_1 = E_2 \omega_2$

(B)  $\frac{\omega_2}{\omega_1} = n^2$

(C)  $\omega_1 \omega_2 = n^2$

(D)  $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

**Ans.**

(B,D)

For first oscillator  
प्रथम दोलक के लिए

$$b = m a \omega_1$$

$$\frac{a}{b} = \frac{1}{m \omega_1} = n^2$$

For Second oscillator  
द्वितीय दोलक के लिए

$$\frac{1}{m \omega_2} = 1$$

$$\frac{\omega_2}{\omega_1} = n^2 \quad \text{Ans. B}$$

$$E_1 = \frac{1}{2} m \omega_1^2 a^2;$$

$$E_2 = \frac{1}{2} m \omega_2^2 R^2$$

$$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} \times n^2 = \frac{\omega_1^2}{\omega_2^2} \times \frac{\omega_2}{\omega_1};$$

$$\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2} \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Ans. D



20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and  $U_0$  are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

[JEE(Advanced) 2015 ; 8/88, -1]

## Column-I

(A)  $U_1(x) = \frac{U_0}{2} \left[ 1 - \left( \frac{x}{a} \right)^2 \right]^2$

(B)  $U_2(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2$

(C)  $U_3(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2 \exp \left[ - \left( \frac{x}{a} \right)^2 \right]$

(D)  $U_4(x) = \frac{U_0}{2} \left[ \frac{x}{a} - \frac{1}{3} \left( \frac{x}{a} \right)^3 \right]$

## Column-II

(P) the force acting on the particle is zero at  $x = a$ .

(Q) the force acting on the particle is zero at  $x = 0$ .

(R) the force acting on the particle is zero at  $x = -a$ .

(S) The particle experiences an attractive force towards  $x = 0$  in the region  $|x| < a$

(T) The particle with total energy  $\frac{U_0}{4}$  can oscillate about the point  $x = -a$ .

इकाई द्रव्यमान का एक कण एक बल के प्रभाव में x-अक्ष पर गति कर रहा है। कण की कुल ऊर्जा संरक्षित है। कॉलम I में कण की स्थितिज ऊर्जाओं के चार संभावित रूप दिये गये हैं (a तथा  $U_0$  स्थिरांक हैं)। कॉलम I में दी गयी स्थितिज ऊर्जाओं का कॉलम II में दिये कथन/कथनों से उचित मिलान कीजिए।

## कॉलम-I

(A)  $U_1(x) = \frac{U_0}{2} \left[ 1 - \left( \frac{x}{a} \right)^2 \right]^2$

(B)  $U_2(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2$

(C)  $U_3(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2 \exp \left[ - \left( \frac{x}{a} \right)^2 \right]$

(D)  $U_4(x) = \frac{U_0}{2} \left[ \frac{x}{a} - \frac{1}{3} \left( \frac{x}{a} \right)^3 \right]$

## कॉलम-II

(P) कण पर कार्य करने वाला बल  $x = a$  पर शून्य है।

(Q) कण पर कार्य करने वाला बल  $x = 0$  पर शून्य है।

(R) कण पर कार्य करने वाला बल  $x = -a$  पर शून्य है।

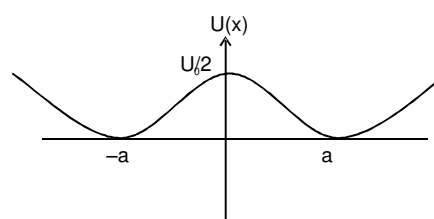
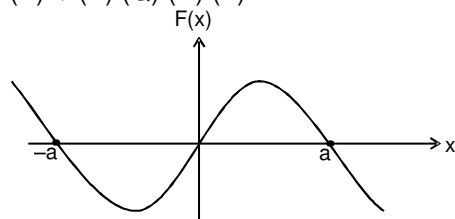
(S) क्षेत्र  $|x| < a$  में कण  $x = 0$  की ओर आकर्षण बल का अनुभव करता है।

(T)  $\frac{U_0}{4}$  कुल ऊर्जा वाला कण  $x = -a$  बिंदु के परितः दोलन कर सकता है।

Ans. (A) → P, Q, R, T (B) → Q, S (C) P, Q, R, S (D) P, R, T

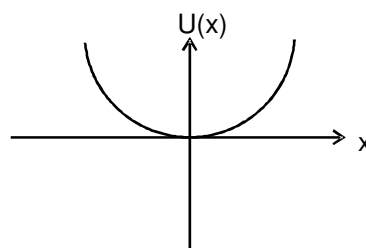
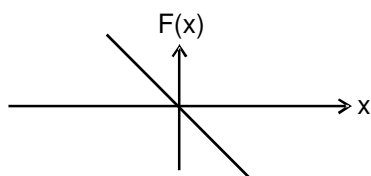
Sol. (A)  $F_x = \frac{-dU}{dx} = -\frac{2U_0}{a^3} [x-a] [x] [x+a]$

(A) → (P) (Q) (R) (T)



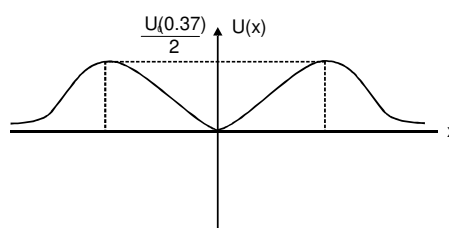
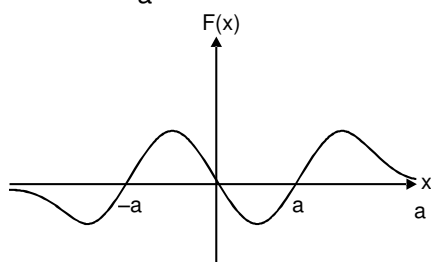


$$(B) F_x = \frac{-dU}{dx} = -U_0 \left( \frac{x}{a} \right)$$



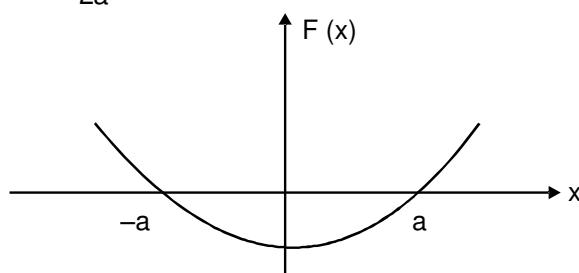
$$(C) F_x = -\frac{dU}{dx}$$

$$= U_0 \frac{e^{-x^2/a^2}}{a^3} [x][x-a][x+a]$$

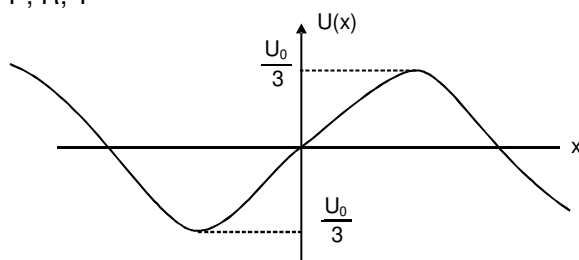


$$(D) F_x = -\frac{dU}{dx} =$$

$$= -\frac{U_0}{2a^3} [(x-a)(x+a)]$$



P, R, T





21. A block with mass  $M$  is connected by a massless spring with stiffness constant  $k$  to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude  $A$  about an equilibrium position  $x_0$ . Consider two cases : (i) when the block is at  $x_0$  ; and (ii) when the block is at  $x = x_0 + A$ . In both the cases, a particle with mass  $m$  ( $< M$ ) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass  $m$  is placed on the mass  $M$  ?

(A\*) The amplitude of oscillation in the first case changes by a factor of  $\sqrt{\frac{M}{m+M}}$ , whereas in the second case it remains unchanged

(B\*) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D\*) The instantaneous speed at  $x_0$  of the combined masses decreases in both the cases

एक द्रव्यमान-रहित स्प्रिंग, जिसका द्रढ़ता गुणांक (stiffness constant)  $k$  है, के एक छोर पर  $M$  द्रव्यमान का एक गुटका जुड़ा है, तथा दूसरे छोर को द्रढ़ दीवार से जोड़ा गया है। यह गुटका एक समतल घर्षण-रहित सतह पर एक संतुलित स्थिति  $x_0$  के गिर्द छोटे आयाम  $A$  से दोलन करता है। यहाँ दो परिस्थितियाँ मानिए : (i) जब गुटका  $x_0$  पर है और (ii) जब गुटका  $x = x_0 + A$  पर है। दोनों परिस्थितियों में द्रव्यमान  $m$  ( $< M$ ) के एक कण को गुटके पर धीरे से इस प्रकार रखा जाता है की वह तुरंत गुटके से चिपक जाता है। कण को गुटके के ऊपर रखने के बाद गति के बारे में निम्नलिखित में से कौनसा/कौनसे कथन सत्य है/हैं ?

(A) पहली परिस्थिति में दोलन का आयाम  $\sqrt{\frac{M}{m+M}}$  भाज्य (factor) से परिवर्तित होता है, जबकि दूसरी परिस्थिति में यह अपरिवर्तित रहता है।

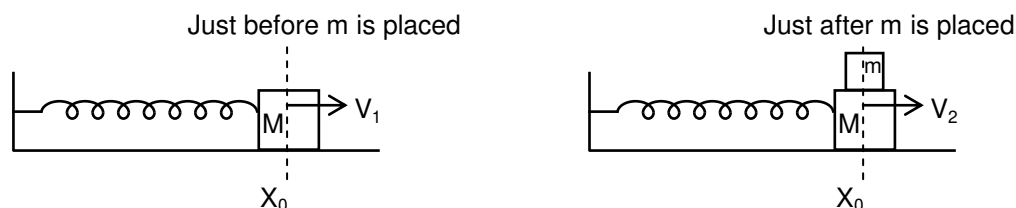
(B) दोनों परिस्थितियों में दोलन का अंतिम समयकाल समान है।

(C) दोनों परिस्थितियों में सम्पूर्ण ऊर्जा कम हो जाती है।

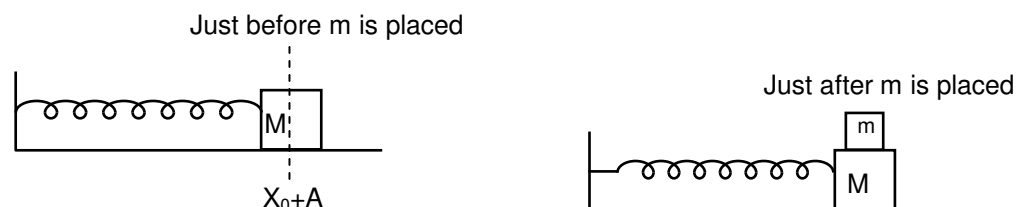
(D) सम्मिलित द्रव्यमानों की  $x_0$  पर तात्क्षणिक गति दोनों परिस्थितियों में कम हो जाती है।

Ans. (ABD)

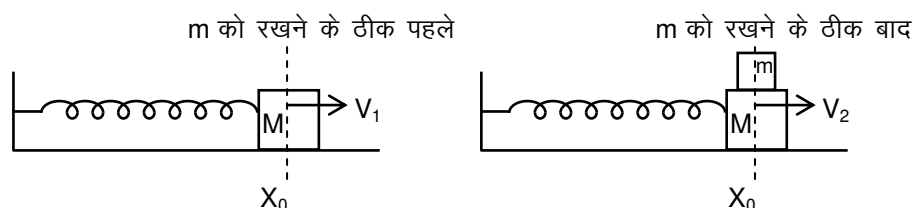
Sol. Case-1



Case-2

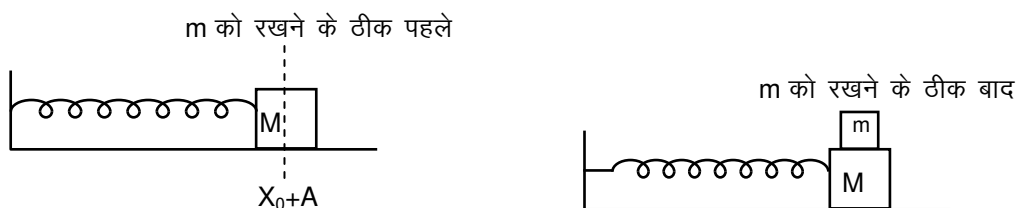


Case-1





## Case-II



In case-1 में

$$MV_1 = (M + m)V_2$$

$$V_2 = \left( \frac{M}{M+m} \right) V_1, \quad \sqrt{\frac{k}{M+m}} A_2 = \left( \frac{M}{M+m} \right) \sqrt{\frac{k}{M}} A_1$$

$$A_2 = \sqrt{\frac{M}{M+m}} A_1$$

In case-2 में

$$A_2 = A_1$$

$$T = 2\pi \sqrt{\frac{M+m}{K}} \text{ in both case. दोनों स्थितियों में}$$

Total energy decreases in first case where as remain same in 2<sup>nd</sup> case.Instantaneous speed at  $x_0$  decreases in both case.

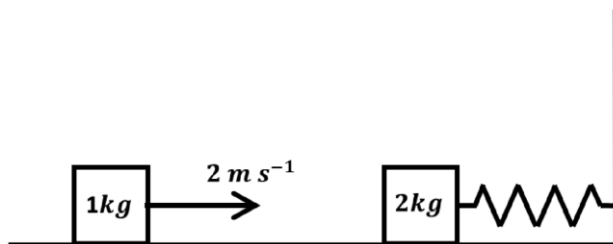
प्रथम स्थिति में कुल ऊर्जा घटती है जबकि द्वितीय स्थिति में नियत रहती है।

 $x_0$  पर तात्क्षणिक चाल दोनों स्थितियों में घटती है।

22. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is  $2.0 \text{ Nm}^{-1}$  and the mass of the block is  $2.0 \text{ kg}$ . Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass  $1.0 \text{ kg}$  moving with a speed of  $2.0 \text{ ms}^{-1}$  collides elastically with the first block. The collision is such that the  $2.0 \text{ kg}$  block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is \_\_\_\_\_.

[JEE (Advanced) 2018, P-1, 3/60]

एक कमानी-गुटका निकाय (spring-block system) एक घर्षण रहित फर्श (frictionless floor) पर विरामावस्था में है, जैसा कि चित्र में दर्शाया गया है। कमानी स्थिरांक (spring constant)  $2.0 \text{ Nm}^{-1}$  है और गुटके का द्रव्यमान (mass)  $2.0 \text{ kg}$  है। कमानी के द्रव्यमान की उपेक्षा कीजिए। शुरुआत में कमानी अतनित (unstretched) अवस्था में है। एक दूसरा गुटका, जिसका द्रव्यमान  $1.0 \text{ kg}$  है और चाल  $2.0 \text{ ms}^{-1}$  है, पहले गुटके से प्रत्यास्थ संघट्ट (elastic collision) करता है। इस संघट्ट के बाद  $2.0 \text{ kg}$  का गुटका दीवार से नहीं टकराता है। जब कमानी संघट्ट के बाद पहली बार अपनी अतनित स्थिति में वापस आती है, तब दोनों गुटकों के बीच की दूरी \_\_\_\_\_ मीटर होगी।



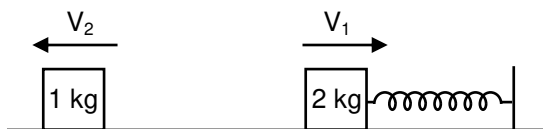


Ans. 2.09

Sol. Just Before Collision टक्कर के पहले



Just After Collision टक्कर के बाद



$$V_1 + V_2 = 2 \quad \dots(1)$$

$$2V_1 - V_2 = 4 \quad \dots(2)$$

$$3V_1 = 4 \Rightarrow V_1 = \frac{4}{3} \quad V_2 = 2 - \frac{4}{3} = \frac{2}{3} \text{ m/s}$$

$$\Delta t = \frac{T}{2} = \frac{2\pi}{2} \sqrt{\frac{2}{2}} = \pi \text{ sec.}$$

$$\text{Distance दूरी} = \pi V_2 = \left(\frac{2}{3}\right) \left(\frac{22}{7}\right) \pi = \frac{44}{21} \pi \text{ m} = 2.09$$

## PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

### भाग - II : JEE (MAIN) / AIEEE (पिछले वर्षों) के प्रश्न

1. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is : [AIEEE 2006; 3/165, -1]

7 मिमी. आयाम से एक सरल आवर्त गति करते हुए एक कण का अधिकतम वेग 4.4 मी/से है। दोलन काल है :

- (1) 100 s (2\*) 0.01 s (3) 10 s (4) 0.1 s

Sol.  $A\omega = v_{\max}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi A}{v_{\max}} = 0.01 \text{ sec.}$$

- 2\*. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency  $\omega$ . The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : [AIEEE 2006; 3/165, -1]

एक क्षैतिज प्लेटफॉर्म पर एक सिक्का रखा है। प्लेटफॉर्म  $\omega$  कोणीय आवृत्ति से ऊर्ध्वाधर सरल आवर्त गति कर रहा है। दोलन का आयाम धीरे-धीरे बढ़ाया जाता है। प्रथम बार सिक्का प्लेटफॉर्म से स्पर्श छोड़ेगा :

- (1\*) at the highest position of the platform (2) at the mean position of the platform

- (3\*) for an amplitude of  $\frac{g}{\omega^2}$  (4) for an amplitude of  $\frac{g^2}{\omega^2}$

- (1\*) प्लेटफॉर्म की उच्चतम अवस्था पर (2) प्लेटफॉर्म की मध्य अवस्था पर

- (3\*)  $\frac{g}{\omega^2}$  के आयाम पर (4)  $\frac{g^2}{\omega^2}$  के आयाम पर

Sol.  $A\omega^2 = g \Rightarrow A = g/\omega^2$





3. The displacement of an object attached to a spring and executing simple harmonic motion is given by  $x = 2 \times 10^{-2} \cos \pi t$  metres. The time at which the maximum speed first occurs is:  
 एक स्प्रिंग से जुड़ी तथा सरल आवर्त गति करने वाली एक वस्तु का विस्थापन  $x = 2 \times 10^{-2} \cos \pi t$  मीटर से दिया जाता है। समय जिस पर पहली बार अधिकतम चाल प्रकट होती है, है

[AIEEE 2007; 3/120, -1]

- (1\*) 0.5 s (2) 0.75 s (3) 0.125 s (4) 0.25 s

Sol.  $|v| = (2 \times 10^{-2}) (\pi) \sin \pi t$

For  $|v|$  to be maximum  $\sin \pi t = 1$

$|v|$  के अधिकतम होने के लिए  $\sin \pi t = 1$

$$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \dots \dots \dots t = \frac{1}{2} \text{ s.}$$

4. A point mass oscillates along the x-axis according to the law  $x = x_0 \cos (\omega t - \pi/4)$ . If the acceleration of the particle is written as  $a = A \cos (\omega t + \delta)$ , then :

[AIEEE 2007; 3/120, -1]

एक बिन्दु द्रव्यमान नियम  $x = x_0 \cos (\omega t - \pi/4)$  के अनुसार x-अक्ष के अनुदिश कम्पन करता है। यदि कण का त्वरण निम्न प्रकार लिखा जाता है,  $a = A \cos (\omega t + \delta)$ , तो

- (1)  $A = x_0, \delta = -\pi/4$  (2)  $A = x_0 \omega^2, \delta = -\pi/4$  (3)  $A = x_0 \omega^2, \delta = -\pi/4$  (4\*)  $A = x_0 \omega^2, \delta = 3\pi/4$

Sol.  $x = x_0 \cos \left( \omega t - \frac{\pi}{4} \right), \quad v = -x_0 \omega \sin \left( \omega t - \frac{\pi}{4} \right)$

$$a = -x_0 \omega^2 \cos \left( \omega t - \frac{\pi}{4} \right), \quad a = x_0 \omega^2 \cos \left( \omega t - \frac{\pi}{4} + \pi \right)$$

$$a = x_0 \omega^2 \cos \left( \omega t + \frac{3\pi}{4} \right)$$

5. Two springs, of force constants  $k_1$  and  $k_2$ , are connected to a mass  $m$  as shown. The frequency of oscillation of mass is  $f$ . If both  $k_1$  and  $k_2$  are made four times their original values, the frequency of oscillation becomes:

[AIEEE 2007; 3/120, -1]

बल नियतांक  $k_1$  तथा  $k_2$  की दो स्प्रिंगें द्रव्यमान  $m$  से चित्रानुसार जुड़ी है। द्रव्यमान के दोलन की आवृत्ति  $f$  है। यदि  $k_1$  तथा  $k_2$  दोनों को प्रारम्भिक मान का चार गुना कर दिया जाए, तो दोलन की आवृत्ति हो जाएगी।



- (1)  $f/2$  (2)  $f/4$  (3)  $4f$  (4\*)  $2f$

Sol.  $f = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}, \quad f_{\text{new}} = \frac{1}{2\pi} \sqrt{\frac{4k_1 + 4k_2}{m}} = 2f.$

6. A particle of mass  $m$  executes simple harmonic motion with amplitude  $a$  and frequency  $\nu$ . The average kinetic energy during its motion from the position of equilibrium to the end is :

[AIEEE 2007; 3/120, -1]

द्रव्यमान  $m$  का एक कण, आयाम  $a$  तथा आवृत्ति  $\nu$  से सरल आवर्त गति करता है। इसकी माध्य स्थिति से एक सिरे तक गति के दौरान औसत गतिज ऊर्जा है :

- (1\*)  $\pi^2 m a^2 \nu^2$  (2)  $\frac{1}{4} m a^2 \nu^2$  (3)  $4\pi^2 m a^2 \nu^2$  (4)  $2\pi^2 m a^2 \nu^2$

Sol.  $K_{\text{av}} = \frac{\int_0^{T/4} \frac{1}{2} m [a \omega \cos(\omega t)]^2 dt}{\int_0^{T/4} dt} = \frac{m a^2 \omega^2}{2 \cdot \frac{T}{4}} \int_0^{T/4} \cos^2(\omega t) dt = \frac{2 m a^2 \omega^2}{T} \cdot \frac{T}{8} = \frac{1}{4} m a^2 \omega^2$

$$= \frac{1}{4} m a^2 (2\pi \nu)^2 = \pi^2 m a^2 \nu^2.$$



- 7.\* If  $x$ ,  $v$  and  $a$  denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period  $T$ , then, which of the following does not change with time?  
 यदि  $x$ ,  $v$  तथा  $a$  क्रमशः आवर्तकाल  $T$  से सरल आवर्त गति करते किसी कण के विस्थापन, वेग तथा त्वरण को निरूपित करते हैं, तब समय के साथ निम्नलिखित में से किसमें परिवर्तन नहीं होता है ? [AIEEE 2007; 3/120, -1]

(1\*)  $\frac{aT}{x}$  (2)  $aT + 2\pi v$  (3)  $\frac{aT}{v}$  (4\*)  $a^2T^2 + 4\pi^2v^2$

Sol.  $x = A \sin \omega t$   
 $v = A \omega \cos \omega t$   
 $a = -A\omega^2 \sin \omega t$

(1)  $\frac{aT}{x} = \left( \frac{-A\omega^2 \sin \omega t}{A \sin \omega t} \right) T = -\omega^2 T = -2\pi\omega$

(2)  $aT + 2\pi v = -A\omega^2 T \sin \omega t + 2\pi A\omega \cos \omega t$

(3)  $\frac{aT}{v} = \frac{-A\omega^2 \sin \omega t \times T}{A\omega \cos \omega t}$

(4)  $a^2T^2 + 4\pi^2 v^2 = + A^2 \omega^4 T^2 \sin^2 \omega t + 4\pi^2 A^2 \omega^2 \cos^2 \omega t = + A^2 \omega^4 \left( \frac{2\pi}{\omega} \right)^2 \sin^2 \omega t + 4\pi^2 A^2 \omega^2 \cos^2 \omega t = 4\pi^2 A^2 \omega^2$

8. A mass  $M$ , attached to a horizontal spring, executes SHM with a amplitude  $A_1$ . When the mass  $M$  passes through its mean position then a smaller mass  $m$  is placed over it and both of them move together with amplitude  $A_2$ . The ratio of  $\left( \frac{A_1}{A_2} \right)$  is : [AIEEE - 2011, 4/120, -1]

एक क्षैतिज कमाने से बँधा एक द्रव्यमान  $M$  आयाम  $A_1$  से सरल आवर्त गति कर रहा है। जब द्रव्यमान  $M$  अपनी माध्य अवस्था से गुजर रहा है, तब एक छोटा द्रव्यमान  $m$  इसके ऊपर रख दिया जाता है और अब दोनों आयाम  $A_2$  से गति करते हैं।  $\left( \frac{A_1}{A_2} \right)$  का अनुपात है :

(1)  $\frac{M}{M+m}$  (2)  $\frac{M+m}{M}$  (3)  $\left( \frac{M}{M+m} \right)^{1/2}$  (4\*)  $\left( \frac{M+m}{M} \right)^{1/2}$

Ans. (4)  
 Sol. C.O.L.M.

रेखीय संवेग संरक्षण से (C.O.L.M.)

$MV_{\max} = (m + M)V_{\text{new}}, V_{\max} = A_1 \omega_1$

$V_{\text{new}} = \frac{MV_{\max}}{(m + M)}$

Now अतः,  $V_{\text{new}} = A_2 \omega_2$

$\frac{MA_1}{(m + M)} \sqrt{\frac{K}{M}} = A_2 \sqrt{\frac{K}{(m + M)}}$

$A_2 = A_1 \sqrt{\frac{M}{(m + M)}}$

$\frac{A_1}{A_2} = \left( \frac{m + M}{M} \right)^{1/2}$

Ans.

9. If a spring of stiffness ' $k$ ' is cut into two parts 'A' and 'B' of length  $\ell_A : \ell_B = 2 : 3$ , then the stiffness of spring 'A' is given by : [AIEEE 2011, 11 May; 4/120, -1]  
 यदि बल नियतांक ' $k$ ' वाली एक स्प्रिंग को लम्बाई  $\ell_A : \ell_B = 2 : 3$ , के अनुपात वाले दो भागों 'A' और 'B' में काटा जाता है, तब स्प्रिंग 'A' का बल नियतांक क्या होगा :

(1)  $\frac{3k}{5}$  (2)  $\frac{2k}{5}$  (3)  $k$  (4\*)  $\frac{5k}{2}$

Ans. (4)



**Sol.**  $\ell_A = \frac{2\ell}{5}$ ,  $\ell_B = \left(\frac{3\ell}{5}\right)$

$$K\ell = K_A\ell_A = K_B\ell_B$$

$$K\ell = K_A \left(\frac{2\ell}{5}\right)$$

$$K_A = \frac{5K}{2} \Rightarrow K_B = \frac{5K}{3}$$

- 10.** Two particles are executing simple harmonic motion of the same amplitude  $A$  and frequency  $\omega$  along the  $x$ -axis. Their mean position is separated by distance  $X_0$  ( $X_0 > A$ ). If the maximum separation between them is  $(X_0 + A)$ , the phase difference between their motion is : **[AIEEE - 2011, 4/120, -1]**

$x$ - अक्ष पर एकसमान आयाम  $A$  और आवृत्ति  $\omega$  से दो कण सरल आवर्त गति कर रहे हैं। उनकी माध्य अवस्था के बीच दूरी  $X_0$  ( $X_0 > A$ ) हैं। यदि उनके बीच अधिकतम दूरी  $(X_0 + A)$  हैं, तब उनकी गति में कलान्तर है:

(1)  $\frac{\pi}{2}$

(2\*)  $\frac{\pi}{3}$

(3)  $\frac{\pi}{4}$

(4)  $\frac{\pi}{6}$

**Ans.** (2)

**Sol.**  $x_1 = A \sin(\omega t + \phi_1)$

$$x_2 = A \sin(\omega t + \phi_2)$$

$$x_1 - x_2 = A \left[ 2 \sin \left[ \omega t + \frac{\phi_1 + \phi_2}{2} \right] \sin \left[ \frac{\phi_1 - \phi_2}{2} \right] \right]$$

$$A = 2A \sin \left( \frac{\phi_1 - \phi_2}{2} \right)$$

$$\frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6} \Rightarrow \phi_1 = \frac{\pi}{3}$$

**Ans.**

- 11.** If a simple pendulum has significant amplitude (up to a factor of  $1/e$  of original) only in the period between  $t = 0$ s to  $t = \tau$ s, then  $\tau$  may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

यदि एक सरल दोलक का समय  $t = 0$ s एवं  $t = \tau$ s, के बीच एक सार्थक आयाम (अपने मूल आयाम के  $1/e$  गुणक तक) रहता है तब  $\tau$  को दोलक का औसत काल कहा जा सकता है। जब दोलक का गोलीय बॉब अपने वेग के समानुपाती मंदन (श्यान कर्षण के कारण) को सहता है, जहाँ 'b' समानुपाती गुणांक है, तब दोलक का औसत आयु काल सेकण्ड में है (यह मान लें कि अवमंदन अल्प है) :

**[AIEEE 2012 ; 4, -1]**

(1)  $\frac{0.693}{b}$

(2) b

(3)  $\frac{1}{b}$

(4\*)  $\frac{2}{b}$

**Ans.** (4)

**Sol.**  $m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt}$

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

here b is damping coefficient यहाँ b मंदक गुणांक है।

This has solution of type

$$x = e^{\lambda t} \text{ substituting this}$$

समीकरण के हल  $x = e^{\lambda t}$  की तरह है। इसका मान रखने पर

$$m\lambda^2 + b\lambda + k = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

on solving for x, we get



x के लिए हल करने पर

$$x = e^{-\frac{b}{2m}t} a \cos(\omega_1 t - \alpha)$$

$$\omega_1 = \sqrt{\omega_0^2 - \lambda^2} \quad \text{where यहाँ } \omega_0 = \sqrt{\frac{k}{m}}$$

$$\lambda = +\frac{b}{2}$$

So, average life अतः औसत आयु काल =  $\frac{2}{b}$

12. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to  $\alpha$  times its original magnitude, where  $\alpha$  equals.

(1) 0.7 (2) 0.81 (3\*) 0.729 (4) 0.6

एक मन्दित दोलित्र का आयाम 5s में अपने मूल परिमाण से घटकर मूल परिमाण का 0.9 गुना हो जाता है। एक और 10s में यह घटकर मूल परिमाण का  $\alpha$  गुना हो जाएगा, जहाँ  $\alpha$  का मान है :

[JEE-Mains 2013, 4/120]

(1) 0.7 (2) 0.81 (3\*) 0.729 (4) 0.6

**Sol.**  $A = A_0 e^{-\frac{bt}{2m}}$   
after 5 second  
5 सेकण्ड पश्चात्  
 $0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \dots(i)$   
After 10 more second  
और 10 सेकण्ड पश्चात्

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \dots(ii)$$

From (i) & (ii)

समीकरण (i) तथा (ii) से

$$A = 0.729 A_0 \quad \text{Hence Ans. (3)}$$

13. A particle moves with simple harmonic motion in a straight line. In first  $\tau$  s, after starting from rest it travels a distance  $a$ , and in next  $\tau$  s it travels  $2a$ , in same direction, then : [JEE MAIN 2014; 4, -1]

(1) amplitude of motion is  $3a$  (2) time period of oscillations is  $8\tau$   
(3) amplitude of motion is  $4a$  (4\*) time period of oscillations is  $6\tau$

एक कण एक सरल रेखा में सरल आवर्त गति से गतिशील है। यह विरामावस्था से प्रारम्भ कर प्रथम  $\tau$  सेकण्ड में दूरी  $a$  और अगले  $\tau$  सेकण्ड में दूरी  $2a$  उसी दिशा में तय करता है। तब :

(1) गति का आयाम  $3a$  है। (2) दोलों का आवर्त काल  $8\tau$  है।  
(3) गति का आयाम  $4a$  है। (4\*) दोलों का आवर्त काल  $6\tau$  है।

**Ans. (4)**

**Sol.**  $x = A \cos \omega t$   
displacement in  $t$  time =  $A - A \cos \omega t$   
 $t$  समय में विस्थापन =  $A - A \cos \omega t$   
for  $t = \tau$  के लिए  $A [1 - \cos \omega \tau] = a$   
for  $t = 2\tau$  के लिए  $A [1 - \cos 2\omega \tau] = 3a$

$$\frac{1 - \cos \omega \tau}{1 - \cos 2\omega \tau} = \frac{1}{3}$$

$$\frac{1 - \cos \omega \tau}{2 \sin^2 \omega \tau} = \frac{1}{3}$$



Say माना  $x = \cos \omega t$

$$\frac{1-x}{2(1-x^2)} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{2(1+x)} = \frac{1}{3}$$

$$\Rightarrow 3 = 2 + 2x \Rightarrow x = \frac{1}{2} = \cos \omega \tau$$

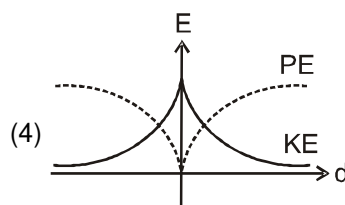
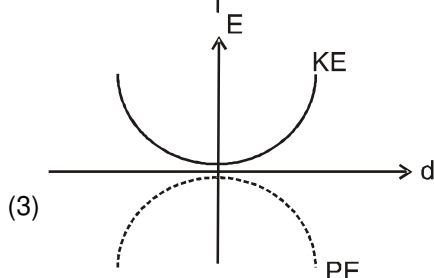
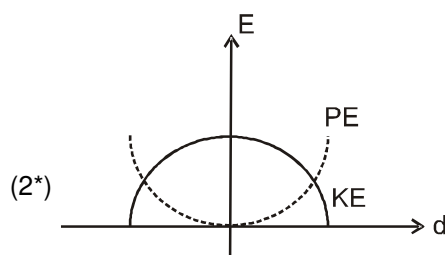
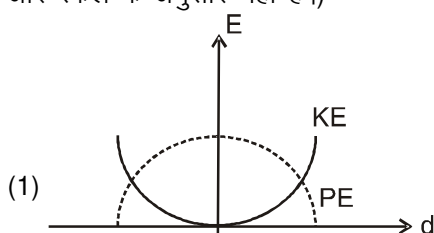
$$A = 2a, \quad \omega \tau = \frac{\pi}{3} \Rightarrow \frac{2\pi}{T} \tau = \frac{\pi}{3} \Rightarrow T = 6\tau$$

14. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement  $d$ . Which one of the following represents these correctly ?

(graphs are schematic and not drawn to scale)

[JEE(Main)-2015; 4/120, -1]

किसी सरल लोलक के लिये, उसके विस्थापन  $d$  तथा उसकी गतिज ऊर्जा के बीच और विस्थापन  $d$  तथा उसकी स्थितिज ऊर्जा के बीच ग्राफ खींचे गये हैं। निम्नांकित में से कौनसा ग्राफ (आलेख) सही है ? (यहाँ ग्राफ केवल व्यवस्था आरेख है और स्केल के अनुसार नहीं है।)



Ans. (2)

Sol. K.E. is maximum at mean position, whereas P.E. is minimum.

At extreme position, K.E. is minimum and P.E. is maximum.

माध्य स्थिति पर गतिज ऊर्जा अधिकतम होगी, जबकि स्थितिज ऊर्जा न्यूनतम होगी .

चरम स्थिति पर गतिज ऊर्जा न्यूनतम होगी, जबकि स्थितिज ऊर्जा अधिकतम होगी .

15. A particle performs simple harmonic motion with amplitude  $A$ . Its speed is trippled at the instant that it is at distance  $2A/3$  from equilibrium position. The new amplitude of the motion is.

एक कण  $A$  आयाम से सरल आवर्त दोलन कर रहा है। जब यह अपने मूल स्थान से पर  $2A/3$  पहुँचता है, तब अचानक इसकी गति तिगुनी कर दी जाती है। तब इसका नया आयाम है

[JEE(Main)-2016; 4/120, -1]

- (1)  $3A$  (2)  $\sqrt{3} A$  (3)  $\frac{7A}{3}$  (4)  $\frac{A}{3}\sqrt{41}$

Ans. (3)



**Sol.**  $v = \omega \sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

$$v = \sqrt{5} \frac{A\omega}{3}$$

$$v_{\text{new}} = 3v = \sqrt{5} A\omega$$

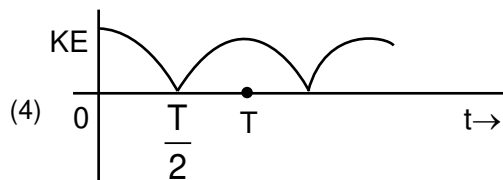
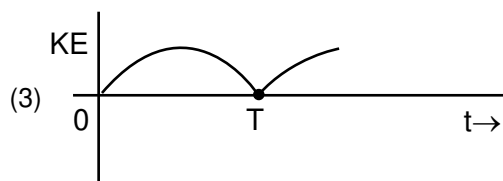
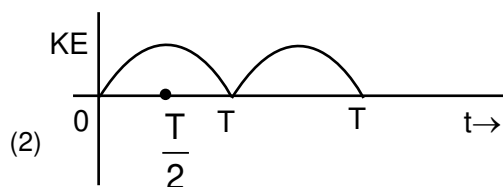
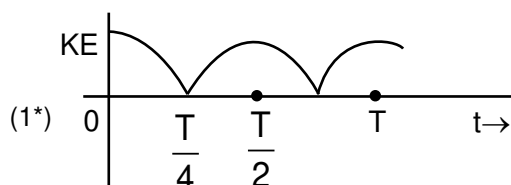
So the new amplitude is given by

अतः नया आयाम होगा

$$v_{\text{new}} = \omega \sqrt{A_{\text{new}}^2 - x^2} \Rightarrow \sqrt{5} A\omega = \omega \sqrt{A_{\text{new}}^2 - \left(\frac{2A}{3}\right)^2}$$

$$A_{\text{new}} = \frac{7A}{3}$$

- 16.** A particle is executing simple harmonic motion with a time period  $T$ . At time  $= 0$ , it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like : **[JEE Main 2017, 4/120, -1]**  
 एक कण आवर्तकाल  $T$  से सरल आवर्त गति कर रहा है। समय  $t = 0$  पर वह साम्यवस्था की स्थिति में है। निम्न में से कौनसा ग्राफ समय के साथ गतिज ऊर्जा का सही दर्शाता है।

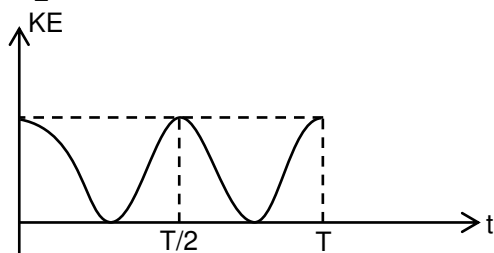


**Ans.** (1)





**Sol.**  $x = A \sin(\omega t + \phi)$   
 $\phi = 0, \pi$   
 $x = \pm A \sin \omega t$   
 $KE = \frac{1}{2} m \omega^2 (A^2 - x^2)$   
 $= \frac{1}{2} K A^2 \cos^2 \omega t$



NOTE : But as per options given, best possible answer will be option (1)

17. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of  $10^{12}/\text{sec}$ . What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avogadro number =  $6.02 \times 10^{23} \text{ gm mole}^{-1}$ ) **[JEE (Main) 2018; 4/120, -1]**

किसी ठोस में चांदी का एक परमाणु  $10^{12}/\text{sec}$  की आवृत्ति से किसी दिशा में सरल आवर्त गति करता है। एक परमाणु को दूसरे परमाणु से जोड़ने वाले बंध का बल नियतांक कितना होगा ? (चांदी का आणविक भार = 108 और अवागाद्री (Avogadro) संख्या =  $6.02 \times 10^{23} \text{ gm mole}^{-1}$ )

- (1) 2.2 N/m                      (2) 5.5 N/m                      (3) 6.4 N/m                      (4\*) 7.1 N/m

**Ans. (4)**

**Sol.**  $T = 2\pi \sqrt{\frac{m}{k}}$   
 $\Rightarrow k = 4\pi^2 m \times f^2$   
 $k = 4\pi^2 \times \frac{108 \times 10^{-3}}{6.02 \times 10^{23}} \times f^2$   
 $= \frac{4 \times \pi^2 \times 108}{6.02} \times \frac{10^{24} \times 10^{-3}}{10^{23}}$   
 $= 7.1 \text{ N/m}$

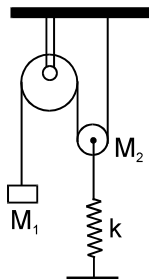


## High Level Problems (HLP)

### SUBJECTIVE QUESTIONS

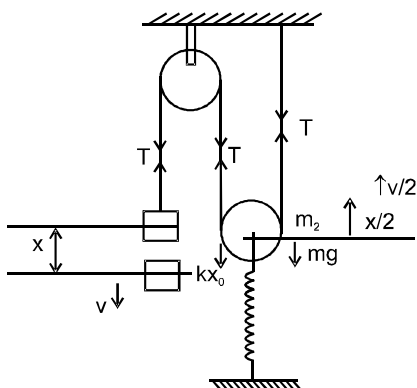
#### विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

- 1.# What would be the period of the free oscillations of the system shown here if mass  $M_1$  is pulled down a little force constant of the spring is  $k$ , mass of fixed pulley is negligible and movable pulley is smooth  
 प्रदर्शित निकाय के मुक्त दोलों का आवर्तकाल क्या होगा यदि  $M_1$  द्रव्यमान को थोड़ा सा नीचे विस्थापित किया जाये।  
 स्प्रिंग का बल नियतांक  $k$  है, स्थिर घिरनी का द्रव्यमान नगण्य है, और चलित घिरनी चिकनी है।



Ans.  $T = 2\pi \sqrt{\frac{M_2 + 4M_1}{k}}$

Sol.



(i) Equilibrium position determination साम्यवस्था स्थिति निर्धारण

$$M_1g = T \rightarrow (\text{from FBD of } M_1) \text{ (} M_1 \text{ के मु. व. रे. से)}$$

$$2T = M_2g + kx_0 \quad (\text{from FBD of } M_2) \text{ (} M_2 \text{ के मु. व. रे. से)}$$

$$\therefore 2M_1g = M_2g + kx_0$$

$$\therefore kx_0 = 2M_1g - M_2g$$

(ii) Displace block  $M_1$  by small disp.  $x$  by वस्तु  $M_1$  को छोटे विस्थापन  $x$  से विस्थापित करने पर

At new displaced position नयी विस्थापित स्थिति

$$-Mgx + \frac{1}{2}M_1V^2 + \frac{1}{2}M_2\left(\frac{v}{2}\right)^2 + M_2g\left(\frac{x}{2}\right) + \frac{1}{2}K\left(x_0 + \frac{x}{2}\right)^2 = C$$

Differentiating equation समीकरण का अवकलन करने पर

$$-M_1g \frac{dx}{dt} + \frac{1}{2}M_1 2v \frac{dv}{dt} + \frac{M_2}{8} 2v \frac{dv}{dt} + \frac{M_2g}{2} \frac{dx}{dt} + \frac{K}{2} 2\left(x_0 + \frac{x}{2}\right) \left(\frac{1}{2} \frac{dx}{dt}\right) = 0$$

$$\Rightarrow -M_1g + M_1a + \frac{M_2a}{4} + \frac{M_2g}{2} + \frac{K}{2}\left(x_0 + \frac{x}{2}\right) = 0 \quad (\text{where जहाँ } a = \frac{dv}{dt})$$

$$\Rightarrow -M_1g + \frac{M_2g}{2} + M_1a + \frac{M_2a}{4} + \frac{Kx_0}{2} + \frac{Kx}{4} = 0 \text{ (from equilibrium साम्यवस्था से)}$$







$$M_1 g + \frac{M_2 g}{2} + \frac{Kx_0}{2} = 0$$

Hence अतः,  $\frac{4M_1 + M_2}{4} a = \frac{-Kx}{4}$

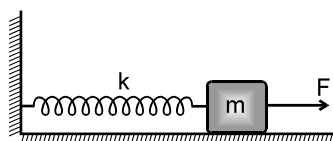
$$\therefore a = -\left(\frac{K}{4M_1 + M_2}\right)x \quad \omega^2 = \frac{K}{(4M_1 + M_2)}$$

$$\omega = \sqrt{\frac{K}{(4M_1 + M_2)}}; \quad T = \frac{2\pi}{\omega}$$

$$\therefore T_2 = 2\pi \sqrt{\frac{4M_1 + M_2}{K}}$$

**2.#** A constant force produces maximum velocity  $V$  on the block connected to the spring of force constant  $K$  as shown in the fig. When the force constant of spring becomes  $4K$ , then find maximum velocity of the block. Assume that initially the spring is in relaxed state.

दिये गये चित्र में  $K$  स्प्रिंग नियतांक की स्प्रिंग से जुड़े ब्लॉक पर नियत बल  $F$  लगाने पर यह ब्लॉक को अधिकतम वेग  $V$  प्रदान करता है। अगर स्प्रिंग का बल नियतांक  $4K$  हो जाए तो ब्लॉक का अधिकतम वेग ज्ञात कीजिए। प्रारम्भ में स्प्रिंग अपनी प्राकृतिक अवस्था में है।



**Ans.  $V/2$**

**Sol(1).** By work energy theorem;

$$W_{\text{ext}} + W_{\text{spring}} = K_f - K_i$$

Let  $x_1, x_2$  be the equilibrium distances of spring from natural length &  $V, V'$  are their velocities at equilibrium positions.

Initially,

$$Fx_1 - \frac{1}{2} kx_1^2 = \frac{1}{2} mv^2 \quad \dots (1)$$

and finally  $Fx_2 - \frac{1}{2} k'x_2^2 = \frac{1}{2} mv'^2 \quad \dots (2)$

In both cases : force applied is same, and velocity becomes maximum when  $F = kx$ . (at equilibrium) (after which the mass will deaccelerate)

$$\therefore F = kx_1 = (4k)x_2$$

$$\Rightarrow x_2 = \frac{x_1}{4} \quad (k' = 4k)$$

Substituting in (2) :

$$fx_2 - \frac{1}{2} k' x_2^2 = \frac{1}{2} mv'^2$$

$$\frac{Fx_1}{4} - \frac{1}{2} (4k) \left(\frac{x_1}{4}\right)^2 = \frac{1}{2} mv'^2$$

$$\Rightarrow \frac{1}{4} [Fx_1 - \frac{1}{2} kx_1^2] = \frac{1}{2} mv'^2 \quad \dots (3)$$

Dividing (3)/(1) ; we get :

$$\frac{1}{4} = \frac{v'^2}{v^2} \quad \Rightarrow \quad v' = \frac{v}{2} \quad \text{Ans.}$$

**Sol(2).**  $V_{\text{max.}} = V = A\omega = \frac{F}{K} \sqrt{\frac{K}{m}}$

$$V' = A'\omega' = \frac{F}{K} \sqrt{\frac{K}{m}} = \frac{V}{2} \quad \text{Ans.}$$





**Sol(1).** कार्य ऊर्जा प्रमेय से

$$W_{\text{ext}} + W_{\text{spring}} = k_f - k_i$$

यहाँ  $x_1, x_2$  क्रमशः प्रारम्भिक व अन्तिम विस्तारण है तथा  $v, v'$  क्रमशः प्रारम्भिक व अन्तिम वेग है।

$$\text{प्रारम्भ में } Fx_1 - \frac{1}{2} kx_1^2 = \frac{1}{2} mv^2 \quad \dots (1)$$

$$\text{अन्त में } Fx_2 - \frac{1}{2} kx_2^2 = \frac{1}{2} mv'^2 \quad \dots (2)$$

दोनों स्थितियों में आरोपित बल समान है। तथा  $F = kx$  पर वेग अधिकतम हो जायेगा (साम्यावस्था पर)

इसके बाद द्रव्यमानों का मंदन होगा

$$\therefore F = kx_1 = (4k)x_2$$

$$\Rightarrow x_2 = \frac{x_1}{4}$$

(2) में प्रतिस्थापन से :

$$\begin{aligned} & \frac{Fx_1}{4} - \frac{1}{2} (4k) \left( \frac{x_1}{4} \right)^2 \\ \Rightarrow & \frac{1}{4} [Fx_1 - \frac{1}{2} kx_1^2] = \frac{1}{2} mv'^2 \quad \dots (3) \end{aligned}$$

(3)/(1) भाजित करने पर हम प्राप्त करते हैं

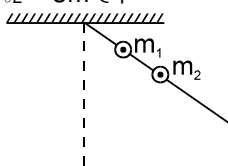
$$\frac{1}{4} = \frac{v'^2}{v^2} \Rightarrow v' = \frac{v}{2} \quad \text{Ans.}$$

**Sol(2).**  $V_{\text{max.}} = V = A\omega = \frac{F}{K} \sqrt{\frac{K}{m}}$

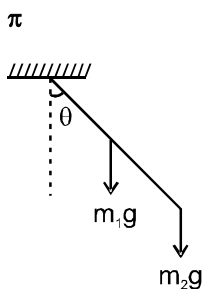
$$V' = A'\omega' = \frac{F}{K} \sqrt{\frac{K}{m}} = \frac{V}{2} \quad \text{Ans.}$$

**3\_#** Two point masses  $m_1$  and  $m_2$  are fixed to a light rod hinged at one end. The masses are at distances  $\ell_1$  and  $\ell_2$  respectively from the hinge. Find the time period of oscillation (small amplitude) of this system in seconds if  $m_1 = m_2$ ,  $\ell_1 = 1\text{m}$ ,  $\ell_2 = 3\text{m}$ .

दो बिन्दु द्रव्यमान  $m_1$  तथा  $m_2$  एक हल्की छड़ पर जुड़े हुए हैं एवं छड़ एक सिरे पर किलकित है।  $m_1$  तथा  $m_2$  किलकित बिन्दु से क्रमशः  $\ell_1$  तथा  $\ell_2$  दूरी पर स्थित हैं। इस निकाय के दोलन का आवर्तकाल सेकण्ड में ज्ञात करो। (आयम अल्प मानिए) यदि  $m_1 = m_2$ ,  $\ell_1 = 1\text{m}$  और  $\ell_2 = 3\text{m}$  है।



**Ans.**  
**Sol.**



Net restoring torque about O O के सापेक्ष कुल प्रत्यानयन बल

$$= m_1g \ell_1 \sin\theta + m_2g \ell_2 \sin\theta = (m_1\ell_1 + m_2\ell_2)g \sin\theta$$

for small oscillation छोटे दोलनों के लिए  $\sin\theta \sim \theta$

$$\Rightarrow \tau_{\text{restoring}} = (m_1\ell_1 + m_2\ell_2)g\theta$$

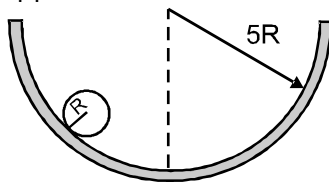


compare with  $\tau = C\theta$  से तुलना करने पर

$$C = (m_1 \ell_1 + m_2 \ell_2)g$$

$$T = 2\pi \sqrt{\frac{I_0}{C}} = 2\pi \sqrt{\frac{(m_1 \ell_1^2 + m_2 \ell_2^2)}{(m_1 \ell_1 + m_2 \ell_2)g}} = \pi$$

4. # A solid sphere (radius = R) rolls without slipping in a cylindrical vessel (radius = 5R). Find the angular frequency of small oscillations of the sphere in  $s^{-1}$ . Take  $R = \frac{1}{14} m$  and  $g = 10 m/s^2$ . (axis of cylinder is fixed and horizontal)
- एक ठोस गोला (त्रिज्या = R) एक बेलन (त्रिज्या = 5R) में बिना फिसले लुढ़कता है। ठोस गोले के अल्प दोलनों का कोणीय आवृत्ति  $s^{-1}$  में ज्ञात करे।  $R = \frac{1}{14} m$  तथा  $g = 10 m/s^2$  लीजिये। (बेलन का अक्ष स्थिर और क्षैतिज है।)



**Ans. 5**

**Sol.** For pure rolling to take place,

$$v = R\omega$$

$\omega'$  = angular velocity of COM of sphere C about O

$$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

$$\therefore \frac{d\omega'}{dt} = \frac{1}{4} \frac{d\omega}{dt} \Rightarrow \alpha' = \frac{\alpha}{4}$$

or  $\alpha = \frac{a}{R}$  for pure rolling

where,  $a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{5g \sin \theta}{7}$

as,  $I = \frac{2}{5} mR^2$

For small  $\theta$ ,  $\sin \theta \approx \theta$ , being restoring in nature.

$$\alpha' = -\frac{5g}{28R} \theta$$

$$\therefore \omega = \sqrt{\frac{\alpha'}{\theta}} = \sqrt{\frac{5g}{28R}} = 5$$

**हल** शुद्ध लोटनी गति के लिए,

$$v = R\omega$$

$\omega'$  = गोले के COM का O के परितः कोणीय वेग

$$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

$$\therefore \frac{d\omega'}{dt} = \frac{1}{4} \frac{d\omega}{dt} \Rightarrow \alpha' = \frac{\alpha}{4}$$

या  $\alpha = \frac{a}{R}$  शुद्ध लोटनी गति के लिए

जहाँ,  $a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{5g \sin \theta}{7}$



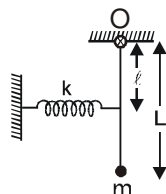
चूँकि  $I = \frac{2}{5} mR^2$

अल्प  $\theta$  के लिए  $\sin \theta \approx \theta$ , प्रकृति में (प्रत्यानयन) है।

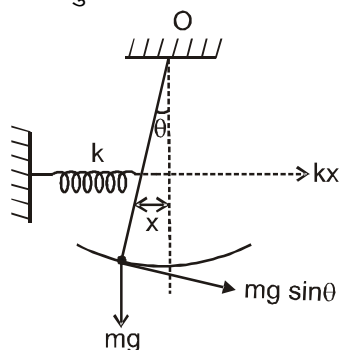
$$\alpha' = -\frac{5g}{28R} \theta \quad \therefore \quad \omega = \sqrt{\left| \frac{\alpha'}{\theta} \right|} = \sqrt{\frac{5g}{28R}} = 5$$

- 5.# A particle of mass  $m$  is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point O. The spring is undeformed when the rod is vertical as shown in fig. If the period of oscillation of the system is  $\pi\sqrt{\frac{L}{n}}$ , when it is slightly displaced from its mean position then find  $n$ . Take  $k = \frac{9mgL}{\ell^2}$  and  $g = 10\text{m/s}^2$ .

एक  $m$  द्रव्यमान का कण नगण्य द्रव्यमान की पतली छड़ के निचले सिरे से लटका है। छड़ का ऊपरी सिरा कागज के तल में O बिन्दु से पारित क्षैतिज अक्ष के सापेक्ष घूर्णन करने के लिए स्वतन्त्र है। चित्रानुसार जब छड़ ऊर्ध्वाधर स्थिति में है तो स्प्रिंग असम्पीड़ित है। यदि इसे साम्यावस्था से थोड़ा-सा विस्थापित करके छोड़ दिया जाये तो कण की गति सरल आवर्त गति है तथा दोलन का आवर्तकाल  $\pi\sqrt{\frac{L}{n}}$  है तो  $n$  ज्ञात करो।  $k = \frac{9mgL}{\ell^2}$  तथा  $g = 10\text{m/s}^2$  लीजिये।



- Ans. 25  
Sol. When the small angular displacement Given to rod जब छड़ को लघु कोणीय विस्थापन दिया जाता है—



for small displacement छोटे विस्थापन के लिये

$$x = \ell \sin \theta$$

$$kx = k\ell \sin \theta$$

$$\text{Torque about point O} = I_O \alpha$$

$$\text{बिन्दु O के सापेक्ष बलाघूर्ण} = I_O \alpha$$

$$(kx)\ell + mg \sin \theta \ell = mL^2 \alpha$$

$$\alpha = \frac{(k\ell^2 + mgL) \sin \theta}{mL^2}$$

For small  $\theta$  लघु  $\theta$  के लिये

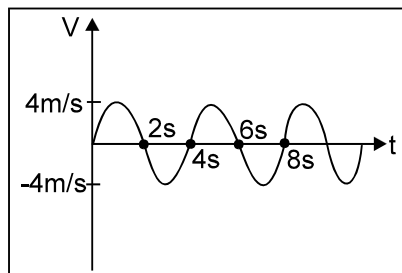
$$\alpha = -\left[ \frac{k\ell^2 + mgL}{mL^2} \right] \theta$$

$$\text{Time period आवर्तकाल } T = 2\pi \sqrt{\frac{mL^2}{k\ell^2 + mgL}} = \pi\sqrt{\frac{L}{25}}$$



- 6.# If velocity of a particle moving along a straight line changes sinusoidally with time as shown in the given graph. Find the average speed over time interval  $t = 0$  to  $t = 2(2n - 1)$  seconds,  $n$  being any positive integer.

अगर सीधी रेखा के अनुदिश गति करते कण का वेग समय के साथ ज्या (sin) फलन के रूप में चित्रानुसार परिवर्तित होता है तो  $t = 0$  से  $t = 2(2n - 1)$  सेकण्ड के दौरान कण की औसत चाल क्या होगी, जहां  $n$  कोई भी धनात्मक पूर्णांक है।



**Ans.**  $\frac{8}{\pi}$  m/s

**Sol. Method - 1**

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{S}{2(2n-1)}$$

Here  $\Delta t = 2(2n - 1) = 4n - 2 = 4(n - 1) + 2$

From the graph it is clear that the Time period  $T = 4$  sec.

$$\therefore \Delta t = (n - 1)T + \frac{T}{2}$$

Total distance travelled in one time period is  $= 4A$

where 'A' is amplitude

$$\therefore \text{Total distance travelled in } \Delta t \text{ is}$$

$$S = (n - 1)4A + 2A = (2n - 1)2A$$

$$\therefore \langle v \rangle = \frac{(2n-1)2A}{2(2n-1)} = A \quad \text{and} \quad \omega A = v_{\max} = 4 \Rightarrow \frac{2\pi}{4} A = 4$$

$$\Rightarrow A = \frac{8}{\pi} \quad \therefore \langle v \rangle = \frac{8}{\pi} \text{ m/s} \quad \text{Ans.}$$

**Method - 2**

It can be observed from the graph, that average speed in time interval  $t = 0$  to  $t = 2$  sec is same as that in intervals  $t = 0$  to  $t = 4$  sec.,  $t = 0$  to  $t = 8$  sec.,  $t = 0$  to  $t = 12$  sec.....

or  $t = 0$  to  $t = 2(2n - 1)$  seconds

The speed as function of time is

$$v = \left| 4 \sin \frac{2\pi}{T} t \right| = \left| 4 \sin \frac{2\pi}{4} t \right| = \left| 4 \sin \frac{\pi t}{2} \right|$$

The average speed in time interval

$t = 0$  to  $t = 2$  sec is

$$\bar{v} = \frac{\int_0^2 v dt}{\int_0^2 dt} = \frac{\int_0^2 4 \sin \frac{\pi t}{2} dt}{2} = \frac{8}{\pi} \text{ m/s}$$

**Sol. Method - 1**

$$\text{औसत चाल} = \frac{\text{कुल तय दूरी}}{\text{लिया गया कुल समय}} = \frac{S}{2(2n-1)}$$

$$\text{यहाँ } \Delta t = 2(2n-1) = 4n-2 = 4(n-1) + 2$$

$$\text{ग्राफ से आवर्त काल } T = 4 \text{ sec.}$$

$$\therefore \Delta t = (n-1)T + \frac{T}{2}$$

$$\text{एक आवर्त काल में कुल तय दूरी} = 4A$$

यहाँ 'A' आयाम है।

$$\therefore \Delta t \text{ समय में कुल तय दूरी}$$

$$S = (n-1)4A + 2A = (2n-1)2A$$

$$\therefore \langle v \rangle = \frac{(2n-1)2A}{2(2n-1)} = A \quad \text{तथा} \quad \omega A = v_{\max} = 4 \Rightarrow \frac{2\pi}{4} A = 4$$

$$\Rightarrow A = \frac{8}{\pi} \quad \therefore \langle v \rangle = \frac{8}{\pi} \text{ m/s} \quad \text{Ans.}$$

**Method - 2**

ग्राफ से देख सकते हैं कि  $t = 0$  से  $t = 2$  सेकण्ड तक औसत चाल अन्तराल  $t = 0$  से  $t = 4$  सेकण्ड  $t = 0$  से  $t = 8$ ,  $t = 0$  से  $t = 12$  सेकण्ड तक की औसत चाल के समान है।

या  $t = 0$  से  $t = 2(2n-1)$  सेकण्ड

समय के फलन में चाल

$$v = \left| 4 \sin \frac{2\pi}{T} t \right| = \left| 4 \sin \frac{2\pi}{4} t \right| = \left| 4 \sin \frac{\pi t}{2} \right|$$

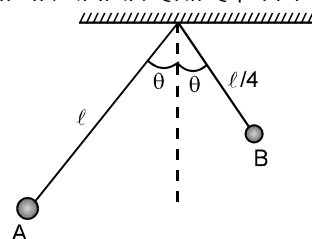
समयान्तराल में औसत चाल

$$t = 0 \text{ से } t = 2 \text{ sec}$$

$$\bar{v} = \frac{\int_0^2 v dt}{\int_0^2 dt} = \frac{\int_0^2 4 \sin \frac{\pi t}{2} dt}{2} = \frac{8}{\pi} \text{ m/s}$$

- 7.#** Two simple pendulums A and B having lengths  $\ell$  and  $\ell/4$  respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle  $\theta$  is very small.

दो सरल लोलको A तथा B की लम्बाई  $\ell$  तथा  $\ell/4$  है को चित्र में प्रदर्शित स्थिति से छोड़ा जाता है। छोड़ने के बाद वह समय ज्ञात करो जब दोनों रस्सीयां पहली बार समान्तर होती है। कोण  $\theta$  बहुत ही अल्प है।

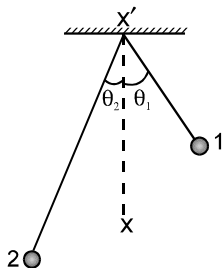


$$\text{Ans. } \frac{\pi}{3} \sqrt{\frac{\ell}{g}}$$





Sol.



The angular position of pendulum 1 and 2 are (taking angles to the right of reference line  $xx'$  to be positive)

सरल लोलकों 1 तथा 2 की कोणीय स्थितियाँ (संदर्भ रेखा  $xx'$  से दायी तरफ कोण धनात्मक है)

$$\theta_1 = \theta \cos\left(\frac{4\pi}{T}t\right) \quad [\text{where यहाँ } T = 2\pi\sqrt{\frac{\ell}{g}}]$$

$$\theta_2 = -\theta \cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

For the strings to be parallel for the first time

पहली बार रस्सीयाँ समान्तर होने पर

$$\theta_1 = \theta_2$$

or या  $\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$

$$\therefore \frac{4\pi}{T}t = 2n\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$$

for  $n = 0$ ,  $t = \frac{T}{2}$  के लिए

for  $n = 1$ ,  $t = \frac{T}{6}, \frac{3T}{2}$  के लिए

$$\therefore \text{Both the pendulum are parallel to each other for the first time after } t = \frac{T}{6} = \frac{\pi}{3} \sqrt{\frac{\ell}{g}} \quad \text{Ans.}$$

$$\therefore \text{पहली बार लोलको के एक दूसरे के समान्तर होने का समय } t = \frac{T}{6} = \frac{\pi}{3} \sqrt{\frac{\ell}{g}} \quad \text{Ans.}$$

8. A particle of mass 'm' is moving in the x-y plane such that its x and y coordinate vary according to the law  $x = a \sin \omega t$  and  $y = a \cos \omega t$  where 'a' and ' $\omega$ ' are positive constants and 't' is time. Find

- (a) equation of the path. Name the trajectory (path)  
(b) whether the particle moves in clockwise or anticlockwise direction  
(c) magnitude of the force on the particle at any time t.

x-y तल में m द्रव्यमान का कण इस प्रकार गतिशील है कि इसके x तथा y निर्देशांक क्रमशः  $x = a \sin \omega t$  तथा  $y = a \cos \omega t$  के अनुसार परिवर्तित होते हैं। यहाँ 'a' तथा ' $\omega$ ' धनात्मक स्थिरांक तथा 't' समय है। ज्ञात करो

- (a) पथ का समीकरण तथा नाम।  
(b) कण वामावर्त दिशा में गति करेगा या दक्षिणावर्त दिशा में गति करेगा ?  
(c) t समय पर कण पर कार्यरत बल ?

Ans.

(a)  $x^2 + y^2 = a^2$ , circle वृत्त

(b) The particle moves in clock wise sense. कण दक्षिणावर्त दिशा में गति करता है।

(c) The magnitude of force बल का परिमाण  $= m \sqrt{a_x^2 + a_y^2} = m\omega^2 a$

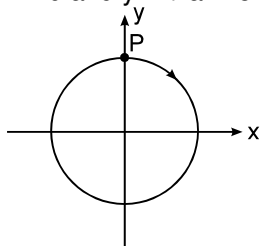


Sol.

$$(a) x^2 + y^2 = a^2 (\sin^2 \omega t + \cos^2 \omega t) = a^2$$

Hence the particle moves in a circle of radius 'a' with centre at origin.

(b) At  $t = 0$  sec,  $x = 0$  and  $y = +a$ . Hence the particle is at P as shown in figure.



As 't' increases 'x' increases and 'y' decreases

$\therefore$  The particle moves in clock wise sense.

(c) The 'x' and 'y' components of acceleration are

$$a_x = \frac{d^2x}{dt^2} = -\omega^2 x ; a_y = \frac{d^2y}{dt^2} = -\omega^2 y$$

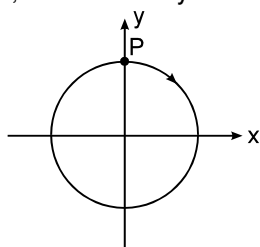
The magnitude of force =  $m \sqrt{a_x^2 + a_y^2} = m\omega^2 a$

Sol.

$$(a) x^2 + y^2 = a^2 (\sin^2 \omega t + \cos^2 \omega t) = a^2$$

अतः कण 'a' त्रिज्या वाले वृत्त पर गति करेगा

(b) अतः  $t = 0$  sec, पर  $x = 0$  और  $y = +a$  है। अतः कण चित्र में P बिन्दु पर है।



जब 't' बढ़ता है तो 'x' बढ़ता है तथा 'y' घटता है

$\therefore$  अतः कण दक्षिणावर्त गति करेगा

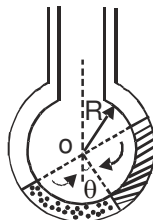
$$(c) \text{ त्वरण के 'x' तथा 'y' घटक } a_x = \frac{d^2x}{dt^2} = -\omega^2 x ; a_y = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$\text{बल का परिणाम} = m \sqrt{a_x^2 + a_y^2} = m\omega^2 a$$

9.#

Two non-viscous, incompressible and immiscible liquids of densities  $\rho$  and  $1.5\rho$  are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in fig. Each liquid occupies one-fourth the circumference of the tube.

दो अश्यान, असंपीड्य तथा अघुलनशील द्रव जिनका घनत्व  $\rho$  तथा  $1.5\rho$  है। R त्रिज्या तथा अल्प अनुप्रस्थ काट क्षेत्रफल की ऊर्ध्वाधर नली में चित्रानुसार भरे हैं। प्रत्येक द्रव नली की चौथाई परिधि में भरा है।



(a) Find the angle  $\theta$  that the radius to the interface makes with the vertical in equilibrium position.

कोण  $\theta$  ज्ञात करो जोकि दोनों द्रव को मिलाने वाली त्रिज्या ऊर्ध्वाधर से बनाती है।

(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

यदि सम्पूर्ण द्रव को साम्यावस्था से अल्प विस्थापित कर दिया जाये तो सिद्ध करो कि परिणामी दोलन सरल आवृत्ति होंगे। इन दोलों का आवर्तकाल ज्ञात करो।



**Resonance®**  
Educating for better tomorrow

Corporate Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) – 324005

Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in

Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSH - 9





**Ans.** (a)  $\tan^{-1} \left( \frac{1}{5} \right)$  (b)  $2\pi \sqrt{\frac{R}{6.11}}$

**Sol.** (a) In equilibrium, pressure of same liquid at same level will be same.

साम्यावस्था में समान स्तर, पर समान तरल का दाब समान होगा

Therefore इसलिए,  $P_1 = P_2$

or या  $P + (1.5 \rho g h_1) = P + (\rho g h_2)$

( $P$  = pressure of gas in empty part of the tube)

( $P$  = खाली नली में गैस का दाब)

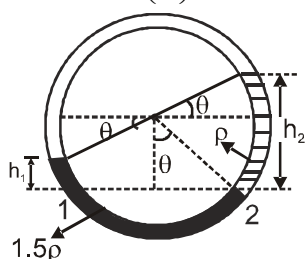
$\therefore 1.5 h_1 = h_2$

$1.5 [R \cos \theta - R \sin \theta] = \rho (R \cos \theta + R \sin \theta)$

or या  $3 \cos \theta - 3 \sin \theta = 2 \cos \theta + 2 \sin \theta$

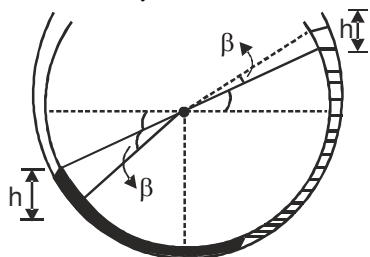
or या  $5 \tan \theta = 1$

$\theta = \tan^{-1} \left( \frac{1}{5} \right)$



(b) When liquids are slightly disturbed by an angle  $\beta$ . Net restoring pressure  $\Delta P = 1.5 \rho g h + \rho g h$ . This pressure will be equal at all sections of the liquid. Therefore, net restoring torque on the whole liquid.

(b) जब तरल को हल्के से  $\beta$  कोण से विक्षेपित करते हैं तो कुल प्रत्यानयन दाबांतर  $\Delta P = 1.5 \rho g h + \rho g h$  है यह दाब तरल में सभी भाग में एक समान होगा। अतः तरल पर कुल प्रत्यानयन बलाघूर्ण



$h = R \sin (\theta + \beta) - R \sin \theta$

$\tau = - (\Delta P) (A) (R)$

or या,  $\tau = - 2.5 \rho g h A R$

$= - 2.5 \rho g A R [R \sin (\theta + \beta) - R \sin \theta]$

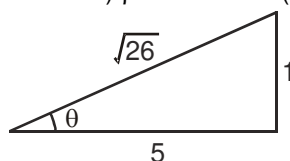
$= - 2.5 \rho g A R^2 [\sin \theta \cos \beta + \sin \beta \cos \theta - \sin \theta]$

Assuming  $\cos \beta = 1$  and  $\sin \beta = \beta$  (given,  $\beta$  is small)

माना  $\cos \beta = 1$  and  $\sin \beta = \beta$  (दिया है,  $\beta$  छोटा है)

$\therefore \tau = - (2.5 \rho A g R^2 \cos \theta) \beta$

or या  $I \alpha = - (2.5 \rho A g R^2 \cos \theta) \beta \dots \dots \dots (1)$



Here, यहाँ

$I = (m_1 + m_2) R^2$

$= \left[ \left( \frac{\pi R}{2} \cdot A \right) \rho + \left( \frac{\pi R}{2} \cdot A (1.5 \rho) \right) \right] R^2$

$= (1.25 \pi R^3 \rho) A$





and और  $\cos\theta = \frac{5}{\sqrt{26}} = 0.98$

Substituting in equation (1), we have  
समीकरण (1) में रखने से प्राप्त होगा

$$\alpha = -\frac{(6.11)\beta}{R} \Rightarrow \text{angular acceleration कोणीय त्वरण } \propto -\text{angular displacement कोणीय विस्थापन}$$

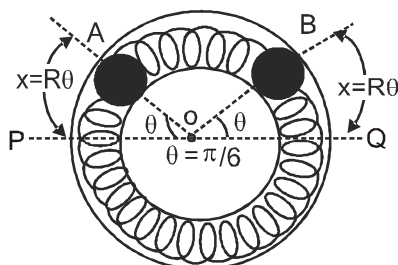
As angular acceleration is proportional to  $-\beta$ , motion is simple harmonic in nature.

अतः कोणीय त्वरण  $-\beta$ , के समानुपाती है, इसलिए गति सरल आवर्त गति होगी।

$$T = 2\pi \sqrt{\frac{\beta}{\alpha}} = 2\pi \sqrt{\frac{R}{6.11}}$$

- 10.#** Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of  $0.06\pi$  metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle  $\theta = \pi/6$  radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest.

समान द्रव्यमान 0.1 kg की दो समरूप गेंदे A व B दो समान द्रव्यमानहीन स्प्रिंगों से जुड़ी है। स्प्रिंग द्रव्यमान निकाय एक वृत्तीय दृढ़ चकनी नली में चित्रानुसार गति कर सकता है। नली क्षैतिज तल में जड़वत् है। गेंदों का केन्द्र 0.06 m त्रिज्या के वृत्त में गति कर सकता है। प्रत्येक स्प्रिंग की सामान्य लम्बाई 0.06π मीटर तथा स्प्रिंग बल नियतांक 0.1 N/m है। प्रारम्भ में दोनों गेंदे  $\theta = \pi/6$  रेडियन कोण से PQ व्यास के सापेक्ष विस्थापित की जाती है तथा स्थिरावस्था से छोड़ी जाती है। (चित्रानुसार)



- (i) Calculate the frequency of oscillation of ball B.

गेंद B के दोलनों की आवृत्ति ज्ञात करो।

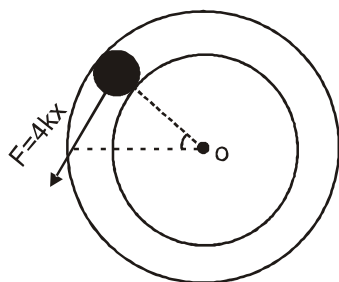
- (ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

गेंद A की चाल ज्ञात करो, जब दोनों गेंद व्यास PQ के दोनों सिरों पर आती है।

- (iii) What is the total energy of the system? निकाय की कुल ऊर्जा क्या है?

**Ans.** (i)  $f = \frac{1}{2\pi} \sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}$  Hz      (ii)  $V = 0.0628$       (iii)  $3.9 \times 10^{-4}$  J

**Sol.** Given – Mass of each block A and B,  $m = 0.1$  kg  
Radius of circle,  $R = 0.06$  m  
दिया गया है – प्रत्येक ब्लॉक A तथा B का द्रव्यमान,  $m = 0.1$  kg  
वृत्त की त्रिज्या,  $R = 0.06$  m  
Natural length of spring  $\ell_0 = \pi R = 0.06\pi$  ( Half circle )  
and spring constant,  $k = 0.1$  N/m  
In the stretched position elongation in each spring  
 $x = R\theta$ .  
Let us draw FBD of A  
स्प्रिंग की सामान्य लम्बाई  $\ell_0 = \pi R = 0.06\pi$  ( आधा वृत्त )  
और स्प्रिंग नियतांक,  $k = 0.1$  N/m  
विस्तारित अवस्था में प्रत्येक स्प्रिंग में विस्तार  
 $x = R\theta$ .  
A का FBD बनाने से



Spring in lower side is stretched by  $2x$  and on upper side compressed by  $2x$ . Therefore, each spring will exert a force  $2kx$  on each ball.

Hence, a restoring force,  $F = 4kx$  will act on A in the direction shown in figure.

Restoring torque of this force about origin

यहाँ नीचे की स्प्रिंग  $2x$  से विस्तारित तथा ऊपर की स्प्रिंग  $2x$  से सम्पीडित हुई है, इसलिए दोनों स्प्रिंग प्रत्येक गेंद पर  $2kx$  बल आरोपित करेगी।

अतः चित्रानुसार प्रत्यानयन बल,  $F = 4kx$  A पर दिखाई गई दिशा में कार्य करेगा।

कुल बिन्दु के परितः प्रत्यानयन बल युग्म का आघूर्ण

$$\tau = -F \cdot R = -(4kx) R = -(4kR\theta) R$$

$$\text{or या } \tau = -4kR^2 \cdot \theta \dots\dots\dots(1)$$

Since,  $\tau \propto -\theta$ , each ball executes angular SHM about origin O.

Eq. (1) can be rewritten as

चूँकि,  $\tau \propto -\theta$ , प्रत्येक गेंद मूल बिन्दु O के परितः कोणिय सरल आवर्त गति करेगी।

समीकरण (1) को निम्न प्रकार लिखा जा सकता है -

$$I\alpha = -4kR^2 \theta$$

$$\text{or या } (mR^2) \alpha = -4kR^2 \theta$$

$$\text{or या } \alpha = -\left(\frac{4k}{m}\right)\theta$$

$$\therefore \text{ Frequency of oscillation दोलन आवृत्ति, } f = \frac{1}{2\pi} \sqrt{\frac{\text{acceleration}}{\text{displacement}}} = \frac{1}{2\pi} \sqrt{\frac{\text{त्वरण}}{\text{विस्थापन}}} = \frac{1}{2\pi} \sqrt{\frac{\alpha}{\theta}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{4k}{m}}$$

Substituting the values, we have मान रखने पर

$$f = \frac{1}{2\pi} \sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi} \text{ Hz}$$

**Alternate वैकल्पिक :**

$$f = \frac{1}{2\pi} \sqrt{\frac{k'}{\mu}}$$

$$\text{where जहाँ } k' = 2k, \mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m}{2}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{4k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi} \text{ Hz}$$

(ii) In stretched position, potential energy of the system is

विस्तारित अवस्था में निकाय की स्थितिज ऊर्जा

$$PE = 2 \left\{ \frac{1}{2} k \right\} \{ 2x \}^2 = 4kx^2 \quad \left( x = \frac{R\pi}{6} \right)$$

and in mean position, both the blocks have kinetic energy only. Hence,

माध्य स्थिति में दोनों ब्लॉक के पास केवल गतिज ऊर्जा होगी। अतः

$$KE = 2 \left\{ \frac{1}{2} mv_{\max}^2 \right\} = mv_{\max}^2$$



From energy conservation

ऊर्जा संरक्षण नियम से

$$PE = KE$$

$$\therefore 4kx^2 = mv_{\max}^2$$

$$\therefore v_{\max} = 2x \sqrt{\frac{k}{m}} = 2R\theta \sqrt{\frac{k}{m}}$$

Substituting the values मान रखने पर

$$v_{\max} = 2 (0.06) \left( \frac{\pi}{6} \right) \sqrt{\frac{0.1}{0.1}} = 0.02\pi$$

or या  $v_{\max} = 0.0628 \text{ m/s}$

(iii) Total energy of the system,  $E = PE$  in stretched position

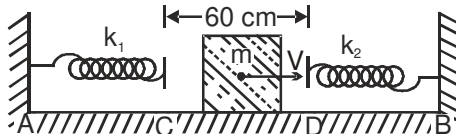
निकाय की कुल ऊर्जा,  $E = PE$  विस्तारित अवस्था में

or या  $= KE$  at mean position  $KE$  माध्य स्थिति पर

$$E = 2 \left( \frac{1}{2} m v_{\max}^2 \right) = (0.1) (0.0628)^2 \text{ J} \quad \text{or} \quad E = 3.9 \times 10^{-4} \text{ J}$$

- 11.#** Two light springs of force constant  $k_1$  and  $k_2$  and a block of mass  $m$  are in one line AB on a smooth horizontal table such that one end of each spring is fixed to rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block. ( $k_1 = 1.8 \text{ N/m}$ ,  $k_2 = 3.2 \text{ N/m}$ ,  $m = 200 \text{ g}$ )

चित्रानुसार  $k_1$  तथा  $k_2$  बल नियतांक की दो हल्की स्प्रिंग तथा  $m$  द्रव्यमान का ब्लॉक चिकनी क्षैतिज टेबल पर एक रेखा AB पर चित्रानुसार स्थित है। दोनों स्प्रिंग के एक सिरे जड़वत् है तथा दूसरा सिरा मुक्त है। दोनों मुक्त सिरों के बीच की दूरी  $CD = 60 \text{ cm}$  है। यदि ब्लॉक को रेखा AB के अनुदिश 120 cm/s का वेग दिया जाये तो ब्लॉक के दोलन का आवर्तकाल ज्ञात करो। ( $k_1 = 1.8 \text{ N/m}$ ,  $k_2 = 3.2 \text{ N/m}$ ,  $m = 200 \text{ g}$ )



**Ans.** 2.82 s

**Sol.** Between C and D block will move with constant speed of 120 cm/s. Therefore, period of oscillation will be (starting from C).

C तथा D के बीच ब्लॉक अचर चाल 120 cm/s. से गतिमान होगा, अतः दोलन का आवर्त काल (C से शुरू करते हुए)

$$T = t_{CD} + \frac{T_2}{2} + t_{DC} + \frac{T_1}{2}$$

Here यहाँ ,  $T_1 = 2\pi \sqrt{\frac{m}{k_1}}$  and और  $T_2 = 2\pi \sqrt{\frac{m}{k_2}}$

and और  $t_{CD} = t_{DC} = \frac{CD}{v} = \frac{60}{120} = 0.5 \text{ s}$

$$\therefore T = 0.5 + \frac{2\pi}{2} \sqrt{\frac{0.2}{3.2}} + 0.5 + \frac{2\pi}{2} \sqrt{\frac{0.2}{1.8}}$$

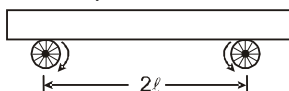
$$(m = 200 \text{ g} = 0.2 \text{ kg})$$

$$T = 2.82 \text{ s}$$



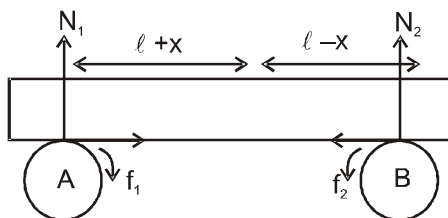
- 12.#** Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass  $M$  is placed on it symmetrically. The friction co-efficient between each wheel and the plank is  $\mu$ . Find the frequency of oscillations, when plank is slightly displaced along its length and released.

चित्रानुसार  $M$  द्रव्यमान की समरूप तख्ता क्षैतिज तथा सममित रूप से दो विपरीत दिशा में नियत कोणीय वेग से किसी बाह्य कारक द्वारा घूमते हुए पहियों पर रखी है। दोनों पहियों तथा तख्ता के मध्य घर्षण गुणांक  $\mu$  है। यदि तख्ता को इसकी लम्बाई के अनुदिश विस्थापित करके छोड़ दिया जाये तो तख्ता की दोलन गति का आवर्तकाल ज्ञात करो।



**Ans.**  $2\pi \sqrt{\frac{\ell}{\mu g}}$

**Sol.** If plank is displaced by  $x$  toward right then  
यदि प्लेट को दायी और  $x$  विस्थापित कर दे तो



Let  $N_1$ ,  $N_2$  and  $f_1$ ,  $f_2$  are Normal and friction force at Point A and B  
by force balance

माना  $N_1$ ,  $N_2$  तथा  $f_1$ ,  $f_2$  अभिलम्ब तथा घर्षण बल बिन्दु A तथा B पर है।

$$N_1 + N_2 = Mg \quad \text{---(1)}$$

and Torque balance बलाघूर्ण संतुलन से

$$Mg(\ell + x) = N_2 \times 2\ell \quad \text{---(2)}$$

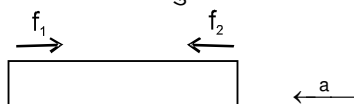
by equation (1) and (2) समीकरण (1) से (2)

$$N_1 = \frac{Mg}{2} - \frac{Mgx}{2\ell} \quad N_2 = \frac{Mg}{2} + \frac{Mgx}{2\ell}$$

So अतः  $f_1 = \mu \left( \frac{Mg}{2} - \frac{Mgx}{2\ell} \right)$

and और  $f_2 = \mu \left( \frac{Mg}{2} + \frac{Mgx}{2\ell} \right)$

F.B.D of M M का मु. व. रे.



$$f_2 - f_1 = Ma$$

$$\mu \cdot \frac{2Mgx}{2\ell} = Ma = - \frac{d^2x}{dt^2} \cdot M$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{-\mu gx}{\ell}$$

$$\Rightarrow \text{Time period आवर्तकाल} = 2\pi \sqrt{\frac{\ell}{\mu g}}$$





13. **The Cubic Potential :** Consider a particle of mass  $m$  moving in one dimension under the influence of potential energy

$$u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$$

Here  $\omega$ ,  $\delta$  and  $\alpha$  are real and positive.

(a) Sketch typical plots of  $u(x)$  and identify extrema if any.

(b) Consider the case where (in appropriate units) we have  $m = 1$ ,  $\omega = \sqrt{2}$ ,  $\alpha = 1$  and  $\delta = 1/2$ . Sketch the potential energy  $u(x)$ . If the total energy of the particle moving in this one-dimensional potential is  $E = 0$  (in same units), discuss the motion of the particle in terms of allowed regions, boundedness and periodicity.

**घनाम विभव (Cubic Potential) :**  $m$  द्रव्यमान का एक कण लेते हैं, जो स्थितिज ऊर्जा  $u(x)$  के प्रभाव में एक विमीय गति करता है।

$$u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$$

यहाँ  $\omega$ ,  $\delta$  तथा  $\alpha$  धनात्मक एवं वास्तविक हैं।

(a)  $u(x)$  का ग्राफ आरेखित करें एवं यदि कोई चरम है तो दर्शाएँ

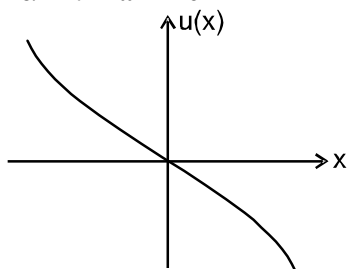
(b) प्रकरण लेवें जहाँ  $m = 1$ ,  $\omega = \sqrt{2}$ ,  $\alpha = 1$  तथा  $\delta = 1/2$  है (उपयुक्त विमा में)। स्थितिज ऊर्जा  $u(x)$  को आरेखित कीजिए। यदि इस एक विमीय विभव में गतिशील कण की कुल ऊर्जा  $E = 0$  है (समान मात्रक में)। अनुमत परास, परिवद्धता एवं आवृत्ति (regions, boundedness and periodicity) के पदों में कण की गति को समझाइये।

**Ans.** (b) between  $x = 0$  and  $x = \frac{3 - \sqrt{3}}{2}$ .  $U$  is  $(-ve)$ . So, K.E. is  $+ve$ .

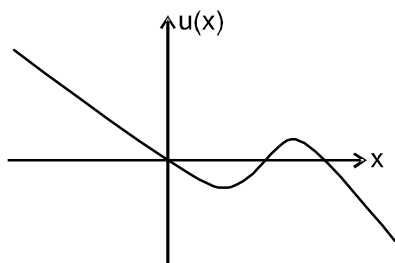
$x = 0$  एवं  $x = \frac{3 - \sqrt{3}}{2}$  में मध्य  $U$  ऋणात्मक है। अतः गतिज ऊर्जा धनात्मक है।

**Sol.** (a)  $U(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$

$$U'(x) = -\alpha x^2 + m\omega^2 x - \delta$$

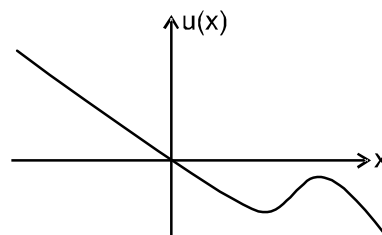
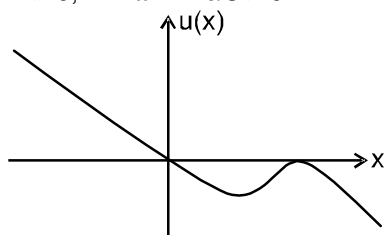


if यदि  $D < 0$ .  $m^2 \omega^4 - 4\alpha \delta < 0$ .





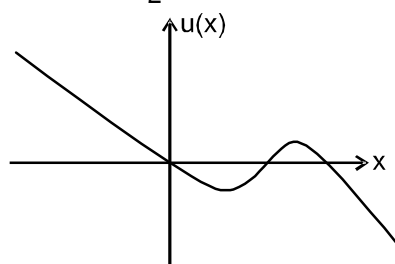
$$D > 0, m^2 \omega^4 - 4\alpha s > 0.$$



$$(b) \quad U(x) = x^2 - \frac{x}{2} - \frac{x^3}{3}$$

$$U'(x) = -x^2 + 2x - \frac{1}{2}$$

$$D = 4 - 4 \times \frac{1}{2} = 2$$



$$U = 0$$

$$\frac{-x}{6}(2x^2 - 6x + 3) = 0$$

$$x = 0, \frac{6 \pm \sqrt{12}}{4}$$

$$x = 0, \frac{3 \pm \sqrt{3}}{2}$$

between  $x = 0$  and  $x = \frac{3 - \sqrt{3}}{2}$ .  $U$  is (-ve). So, K.E. is +ve.

$x = 0$  एवं  $x = \frac{3 - \sqrt{3}}{2}$  में मध्य  $U$  ऋणात्मक है। अतः गतिज ऊर्जा धनात्मक है।

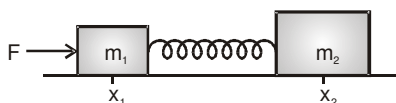
So, this the only allowed region.

अतः केवल यह अनुमत परिक्षेत्र है।

The particle will oscillate between these two points periodically.

कण इन दो बिन्दुओं के मध्य आवर्त रूप से दौलन करेगा।

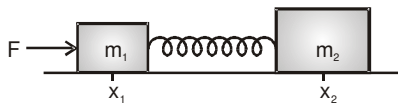
14. Two blocks of masses  $m_1 = 1.0$  kg and  $m_2 = 2.0$  kg are connected by a massless elastic spring and are at rest on a smooth horizontal surface with the spring at its natural length. A horizontal force of constant magnitude  $F = 6.0$  N is applied to the block  $m_1$  for a certain time  $t$  in which  $m_1$  suffers a displacement  $\Delta x_1 = 0.1$  m and  $\Delta x_2 = 0.05$  m. Kinetic energy of the system with respect to center of mass is 0.1 J. The force  $F$  is then withdrawn.



- Calculate  $t$ .
- Calculate the speed and the kinetic energy of the center of mass after the force is withdrawn.
- Calculate the energy stored in the system



$m_1 = 1.0 \text{ kg}$  व  $m_2 = 2.0 \text{ kg}$  द्रव्यमान के दो ब्लॉक द्रव्यमानहीन प्रत्यास्थ स्प्रिंग द्वारा जड़े हुए तथा स्प्रिंग की प्राकृतिक लम्बाई में ये ब्लॉक प्रारम्भ में विरामावस्था में चिकनी क्षैतिज सतह पर रखे हुए हैं।  $m_1$  द्रव्यमान के एक ब्लॉक पर निश्चित समय  $t$  के लिए एक नियत परिमाण का क्षैतिज बल  $F = 6.0 \text{ N}$  आरोपित किया जाता है, जिससे  $m_1$  में विस्थापन  $\Delta x_1 = 0.1 \text{ m}$  तथा  $\Delta x_2 = 0.05 \text{ m}$  होते हैं। द्रव्यमान केन्द्र के सापेक्ष निकाय की गतिज ऊर्जा  $0.1 \text{ J}$  है। इसके पश्चात् बल  $F$  हटा दिया जाता है।



(a)  $t$  की गणन कीजिए।

(b) बल हटाने के बाद द्रव्यमान केन्द्र की चाल तथा गतिज ऊर्जा की गणना कीजिए।

(c) निकाय में संचित ऊर्जा की गणना कीजिए।

**Ans.**

(a)  $0.26 \text{ s}$

(b)  $0.52 \text{ ms}^{-1}$ ,  $0.40 \text{ J}$

(c)  $0.20 \text{ J}$

**Sol.**

(a)  $t = \sqrt{\frac{2(m_1\Delta x_1 + m_2\Delta x_2)}{F}} = 0.26 \text{ s}$

(b) Speed चाल =  $\frac{\sqrt{2F(m_1\Delta x_1 + m_2\Delta x_2)}}{m_1 + m_2} = 0.52 \text{ ms}^{-1}$

Kinetic energy गतिज ऊर्जा =  $\frac{F(m_1\Delta x_1 + m_2\Delta x_2)}{m_1 + m_2} = 0.40 \text{ J}$

(c) Kinetic energy w.r.t. CM + Energy stored =  $\frac{m_2(\Delta x_1 - \Delta x_2)}{m_1 + m_2} F = 0.20 \text{ J}$

द्रव्यमान केन्द्र के सापेक्ष गतिज ऊर्जा + संचित ऊर्जा =  $\frac{m_2(\Delta x_1 - \Delta x_2)}{m_1 + m_2} F = 0.20 \text{ J}$

संचित ऊर्जा =  $0.10 \text{ J}$





## HANDOUT

## SIMPLE HARMONIC MOTION

## DAMPED SIMPLE HARMONIC MOTION

We know that the motion of a simple pendulum, swinging in air, dies out eventually. Why does it happen? This is because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually. The pendulum is said to execute damped oscillations.

In damped oscillations, the energy of the system is dissipated continuously; but, for small damping, the oscillations remain approximately periodic. The dissipating forces are generally the frictional forces. To understand the effect of such external forces on the motion of an oscillator. Let us consider a system as shown in figure below, here a block of mass  $m$  connected to an elastic spring of spring constant  $k$  oscillates vertically. If the block is pushed down a little and released, its angular frequency of oscillation is  $\omega$  ( $\omega = \sqrt{\frac{k}{m}}$ ). However, in practice, the

surrounding medium (air) will exert a damping force on the motion of the block and the mechanical energy of the block-spring system will decrease. The energy loss will appear as heat of the surrounding medium (and the block also)

The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping will be much greater and the dissipation of energy much faster. The damping force is generally proportional to velocity of the bob. [Remember Stokes' Law] and acts opposite to the direction of velocity. If the damping force is denoted by  $F_d$ , we have

$$\vec{F}_d = -b\vec{V}$$

where the positive constant  $b$  depends on characteristics of the medium (viscosity, for example) and the size and shape of the block, etc. This equation is usually valid only for small velocity. When the mass  $m$  is attached to the spring and released, the spring will elongate a little and the mass will settle at some height. This position, shown by  $O$  in Figure 1, is the equilibrium position of the mass. If the mass is pulled down or pushed up a little, the restoring force on the block due to the spring is  $\vec{F}_s = -k\vec{x}$  where  $\vec{x}$

is the displacement of the mass from its equilibrium position. Thus, the total force acting on the mass at any time  $t$ , is  $\vec{F} = -k\vec{x} - b\vec{v}$ . If  $a(t)$  is the acceleration of mass at time  $t$ , then by Newton's Law of Motion applied along the direction of motion, we have  $m a(t) = -kx(t) - b v(t)$

Here we have dropped the vector notation because we are discussing one-dimensional motion. Using the first and second derivatives of  $x(t)$  for  $v(t)$  and  $a(t)$  respectively, we have

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \quad \dots(1)$$

Equation 1 is a second-order linear differential equation and its auxiliary equation is  $mr^2 + br + k = 0$ . The roots are

$$r_1 = \frac{-b + \sqrt{b^2 - 4mk}}{2m} \quad r_2 = \frac{-b - \sqrt{b^2 - 4mk}}{2m} \quad \dots(2)$$

We need to discuss three cases.

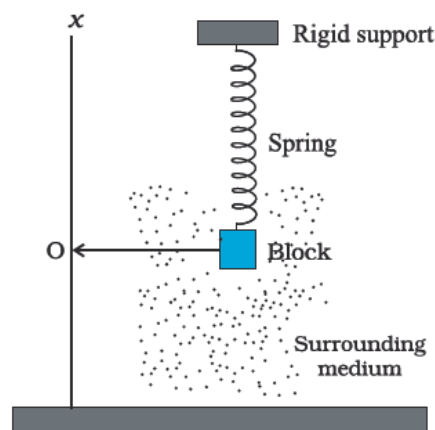
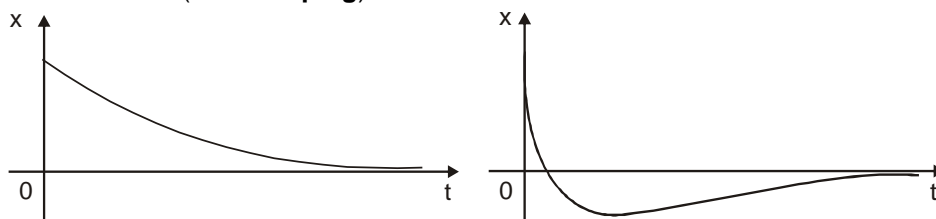


Figure 1: The viscous surrounding medium exerts a damping force on an oscillating spring, eventually bringing it to rest.

**CASE I :  $b^2 - 4mk > 0$  (overdamping)**

In this case  $r_1$  and  $r_2$  are distinct real roots and  $x = b_1 e^{r_1 t} + b_2 e^{r_2 t}$

Since  $b$ ,  $m$ , and  $k$  are all positive, we have  $\sqrt{b^2 - 4mk} < b$ , so the roots  $r_1$  and  $r_2$  given by Equation 2 must both be negative. This shows that  $x \rightarrow 0$  as  $t \rightarrow \infty$ . Typical graphs of  $x$  as a function of  $t$  are shown in above figure. Now Notice that oscillations do not occur. (It's possible for the mass to pass through the equilibrium position once but only once.) This is because  $b^2 > 4mk$  means that there is a strong damping force (high-viscosity oil or grease compared with a weak spring or small mass).

**CASE II :  $b^2 - 4mk = 0$  (critical damping)**

This case corresponds to equal roots

$$r_1 = r_2 = -\frac{b}{2m}$$

and the solution is given by

$$x = (b_1 + b_2 t) e^{-(b/2m)t}$$

It is similar to Case I, and typical graphs resemble those in above figure, but the damping is just sufficient to suppress vibrations. Any decrease in the viscosity of the fluid leads to the vibrations of the following case.

**CASE III :  $b^2 - 4mk < 0$  (underdamping)**

Here the roots are complex:

$$\left. \begin{matrix} r_1 \\ r_2 \end{matrix} \right\} = -\frac{b}{2m} \pm \omega' i$$

$$\text{Where } \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad \dots(3)$$

The solution is given by

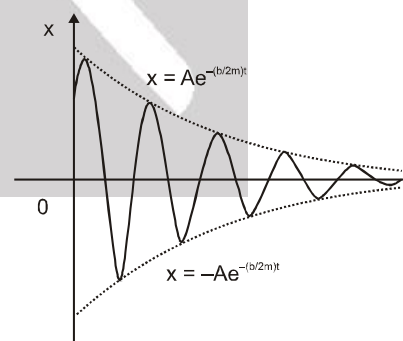
$$x = e^{-(b/2m)t} (b_1 \cos \omega' t + b_2 \sin \omega' t) = A e^{-(b/2m)t} \cos(\omega' t + \phi)$$

We see that there are oscillations the are damped by the factor  $e^{-(b/2m)t}$ . Since  $b > 0$  and  $m > 0$ , we have  $-(b/2m) < 0$  so  $e^{-(b/2m)t} \rightarrow 0$  as  $t \rightarrow \infty$ . This implies that  $x \rightarrow 0$  as  $t \rightarrow \infty$ ; that is, the motion decays to 0 as time increases. A graph is shown in figure.

Now the mechanical energy of the undamped oscillator is  $\frac{1}{2} k A^2$ . For a damped oscillator, the amplitude is not constant but depends on time. For small damping, we may use the same expression but regard the amplitude as  $A e^{-bt/2m}$ .

$$E(t) = \frac{1}{2} k A^2 e^{-bt/m} \quad \dots(4)$$

Equation shows that the total energy of the system decreases exponentially with time. Note that small damping means that is  $\left(\frac{b}{2m}\right)$  much less than  $\omega$ . Of course, as expected, if we put  $b = 0$ , all equations of a damped oscillator in this section reduce to the corresponding equations of an undamped oscillator.



**Quality Factor or Q value:**

Practical applications require consideration of the **quality** of the oscillator,  $Q$ , which specifies the ratio of total energy,  $E$ , to the energy loss,  $\Delta E$ , over one complete oscillation period,  $T$ :

$$Q = 2\pi \frac{E}{|\Delta E|}$$

$$Q = \frac{\omega'}{2\omega_\gamma}$$

where  $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$  and  $\omega_\gamma = \frac{b}{2m}$

In the limit of zero damping, the oscillator experiences no energy loss, and  $Q \rightarrow \infty$ . In the limit of small damping, the quality of the oscillator can be approximated by

$$Q \approx \frac{\omega}{2\omega_\gamma}$$

Combining these two results provides a handy formula for the energy loss during a complete oscillation period of weakly damped motion:

$$|\Delta E| = E \frac{2\pi}{Q} = E \frac{4\pi\omega_\gamma}{\omega} \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

**Example 1.** For the damped oscillator shown in Figure the mass  $m$  of the block is 200 g,  $k = 90 \text{ N m}^{-1}$  and the damping constant  $b$  is  $40 \text{ g s}^{-1}$ . Calculate (a) the period of oscillation, (b) time taken for its amplitude of vibrations to drop to half of its initial value and (c) the time taken for its mechanical energy to drop to half its initial value.

**Answer :** (a) The time period  $T$  from Equation. is given by

$$T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}} = 0.3 \text{ sec}$$

(b) Now, from equation, the time,  $T_{1/2}$  for the amplitude drop half of its initial value is given by

$$A(t) = Ae^{-bt/2m}$$

$$\frac{A}{2} = Ae^{-\frac{bT_{1/2}}{2m}} \Rightarrow T_{1/2} = \frac{2m \ln 2}{b} = 6.93 \text{ s}$$

(c) For calculating the time,  $t_{1/2}$ , for its mechanical energy to drop to half its initial value we make use of equation from this equation we have.

$$E(t_{1/2}) = \frac{E(0)}{2} \Rightarrow E(0)e^{-bt/m} = \frac{E(0)}{2}$$

$$t_{1/2} = \frac{(\ln 2)m}{b} = \frac{6.93}{2}$$

This is just half of the decay period for amplitude. This is not surprising, because, according to Equation 4, energy depends on the square of the amplitude. Notice that there is a factor of 2 in the exponents of the two exponentials.

**Example 2.** Show that the system  $\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x = 0$  is underdamped, find its damped angular frequency

**Solution :**

$$m = 1, b = 1, k = 3$$

$$b^2 < 4mk$$

So, the system is underdamped

$$\omega_d = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} = \frac{\sqrt{11}}{2} \text{ s}^{-1}$$



## FORCED OSCILLATIONS AND RESONANCE

When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency  $\omega$ , and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called force or driven oscillations. We consider the

case when the external force is itself periodic, with a frequency  $\omega_d$  called the driven frequency. A most important fact of forced periodic oscillations is that the system oscillates not with its natural frequency  $\omega$ , but at the frequency  $\omega_d$  of the external agency; the free oscillations die out due to damping. A most familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. Suppose an external force  $F(t)$  of amplitude  $F_0$  that varies periodically with time is applied to a damped oscillator. Such a force can be represented as,

$$F(t) = F_0 \cos \omega_d t$$

The motion of a particle under the combined action of a linear restoring force, damping force and a time dependent driving force represented by Equation is given by,

$$m a(t) = -k x(t) - b v(t) + F_0 \cos \omega_d t$$

Substituting  $d^2x/dt^2$  for acceleration in Equation and rearranging it, we get

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos \omega_d t \quad \dots(5)$$

This is the equation of an oscillator of mass  $m$  on which a periodic force of (angular) frequency  $\omega_d$  is applied. The oscillator initially oscillates with its natural frequency  $\omega$ . When we apply the external periodic force, the

oscillations with the natural frequency die out, and then the body oscillates with the (angular) frequency of the external periodic force. Its displacement, after the natural oscillations die out, is given by

$$x(t) = A \cos (\omega_d t + \phi) \quad \dots(6)$$

where  $t$  is the time measured from the moment when we apply the periodic force. The amplitude  $A$  is a function of the forced frequency  $\omega_d$  and the natural frequency  $\omega$ . Analysis shows that it is given by

$$A = \frac{F_0}{\sqrt{m^2 (\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}} \quad \dots(7)$$

and  $\tan \phi = \frac{-v_0}{\omega_d x_0}$

where  $m$  is the mass of the particle and  $v_0$  and  $x_0$  are the velocity and the displacement of the particle at time  $t = 0$ , which is the moment when we apply the periodic force. Equation (7) shows that the amplitude of the forced oscillator depends on the (angular) frequency of the driving force. We can see a different behaviour

of the oscillator when  $\omega_d$  is far from  $\omega$  and when it is close to  $\omega$ . We consider these two cases.

- (a) Small Damping, Driving Frequency far from Natural Frequency : In this case,  $\omega_d b$  will be much smaller than  $m(\omega^2 - \omega_d^2)$ , and we can neglect that term. Then Equation. (7) reduces to

$$A = \frac{F_0}{m(\omega^2 - \omega_d^2)} \quad \dots(8)$$

Figure (3) shows the dependence of the displacement amplitude of an oscillator on the angular frequency of the driving force for different amounts of damping present in the system. It may be noted that in all the cases the amplitude is greatest when  $\omega_d / \omega = 1$ . The curves in this figure show that smaller the damping, the taller and narrower is the resonance peak.

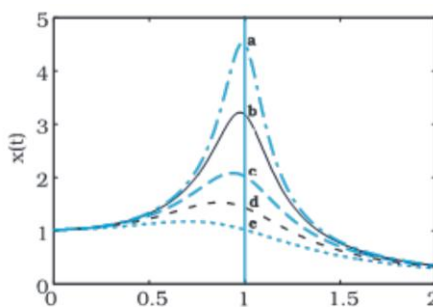


Figure. 3 The graphs illustrate equation (7).  
The resonant amplitude ( $\omega = \omega_d$ ) decreases with increasing damping.

If we go on changing the driving frequency, the amplitude tends to infinity when it equals the natural frequency. But this is the ideal case of zero damping, a case which never arises in a real system as the damping is never perfectly zero. You must have experienced in a swing that when the timing of your push exactly matches with the time period of the swing, your swing gets the maximum amplitude. This amplitude is large, but not infinity, because there is always some damping in your swing. This will become clear in the case (b).

- (b) Driving Frequency Close to Natural Frequency : If  $\omega_d$  is very close to  $\omega$ ,  $m(\omega^2 - \omega_d^2)$  would be much less than  $\omega_d b$ , for any reasonable value of  $b$ , then Equation. (7) reduces to

$$A = \frac{F_0}{\omega_d b}$$

This makes it clear that the maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called resonance. In our daily life we encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realised that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

**Example 3.** A particle of mass  $m$  is attached to a spring (of spring constant  $k$ ) and has a natural angular frequency  $\omega_0$ . An external force  $F(t)$  proportional to  $\cos \omega t$  ( $\omega \neq \omega_0$ ) is applied to the oscillator. The time displacement of the oscillator will be proportional to :

- (1)  $\frac{m}{\omega_0^2 - \omega^2}$       (2)  $\frac{1}{m(\omega_0^2 - \omega^2)}$       (3)  $\frac{1}{m(\omega_0^2 + \omega^2)}$       (4)  $\frac{m}{\omega_0^2 + \omega^2}$

**Solution :**  $x(t) = A \cos(\omega_d t + \phi_0)$

where  $A = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}}$

for small damping

$$A \approx \frac{F_0}{[m(\omega^2 - \omega_d^2)]} \Rightarrow x(t) = \left[ \frac{F_0}{m(\omega^2 - \omega_d^2)} \right] \cos(\omega_d t + \phi_0) \quad \text{Ans. (2)}$$

**Example. 4** In forced oscillation of a particle, the amplitude is maximum for a frequency  $\omega_1$  of the force, while the energy is maximum for a frequency  $\omega_2$  of the force, then :

- (1)  $\omega_1 = \omega_2$       (2)  $\omega_1 > \omega_2$   
(3)  $\omega_1 < \omega_2$  when damping is small and  $\omega_1 > \omega_2$  when damping is large  
(4)  $\omega_1 < \omega_2$

**Solution :** Amplitude and energy both are maximum at resonance, when driving frequency is equal to the natural frequency of oscillation. Hence  $\omega_1 = \omega_2 = \omega_0$

**Ans. (1)**



**Example 5.** If a simple pendulum has significant amplitude (up to a factor of  $1/e$  of original) only in the period between  $t = 0$  s to  $t = \tau$  s, then  $\tau$  may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

(1)  $\frac{0.693}{b}$

(2) b

(3)  $\frac{1}{b}$

(4)  $\frac{2}{b}$

**Solution:**

$$A(t) = A_0 e^{-bt/2m}$$

$$\frac{A_0}{e} = A_0 e^{-bt/2m} \Rightarrow \tau = \frac{2m}{b}$$

In the given question  $\frac{b}{m}$  is given as b.  $\left( \because a = \frac{-kx}{m} - bv \right)$  **Ans (4)**

**Example 6.** The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to  $\alpha$  times its original magnitude, where  $\alpha$  equals.

(1) 0.7

(2) 0.81

(3) 0.729

(4) 0.6

**Solutions :**

$$A(t) = A_0 e^{-bt/2m}$$

Given  $A(5) = 0.9 A_0$

$$0.9 A_0 = A_0 e^{-b(5)/2m}$$

$$A(15) = \alpha A_0$$

$$\alpha A_0 = A_0 e^{-b(15)/2m} \Rightarrow$$

$$\alpha = (0.9)^3 = 0.729$$

**Ans (3)**

**Example 7.** Determine whether the system is underdamped, overdamped or critically damped.

(i)  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$

(ii)  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$

**Solution :**

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

If  $b^2 < 4mk$  then the system is underdamped

If  $b^2 > 4mk$  then the system is overdamped

If  $b^2 = 4mk$  then the system is critically damped

(i) overdamped (ii) Critically damped